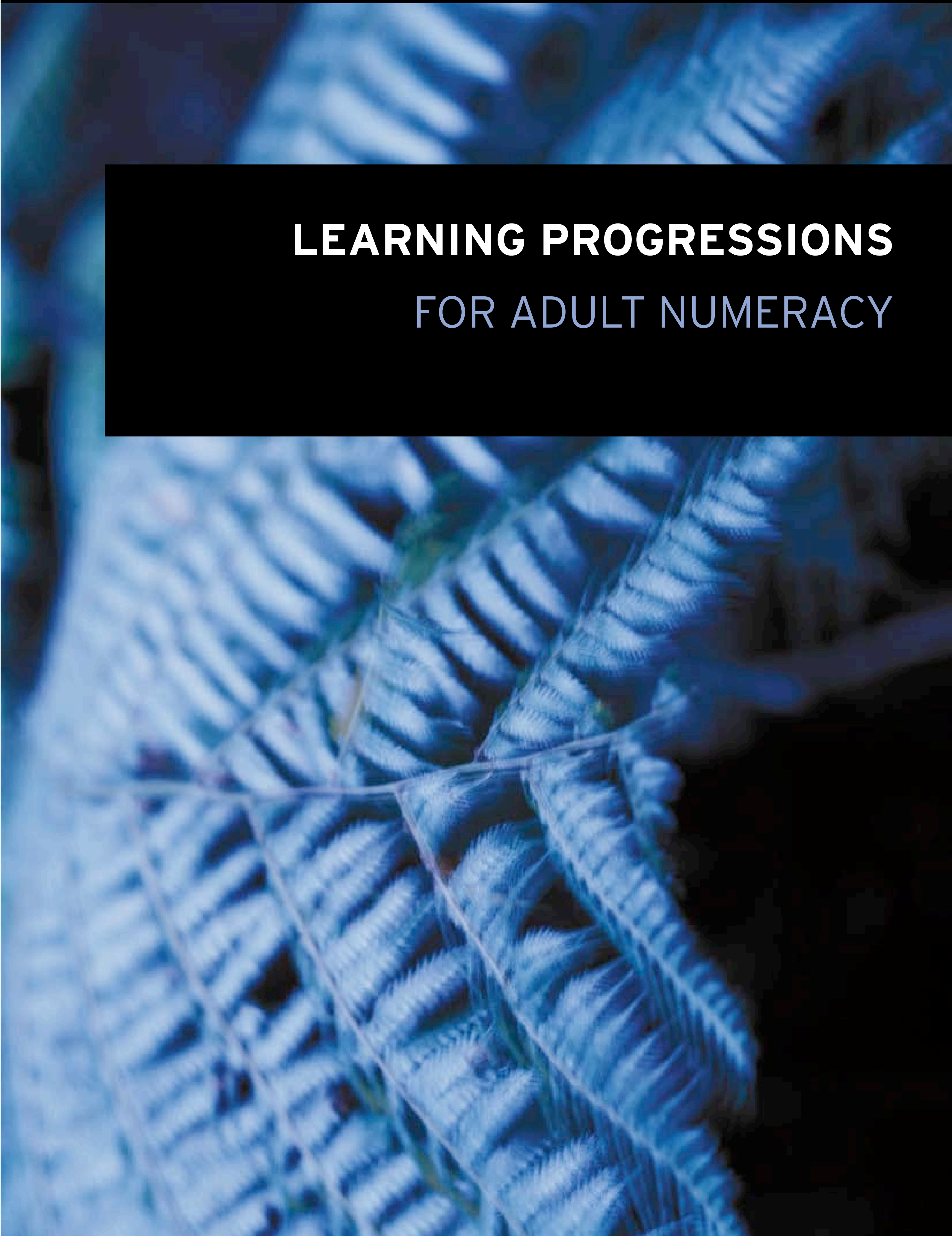
Tertiary Education Commission
Te Amorangi Mātauranga Matua

LEARNING PROGRESSIONS

FOR ADULT NUMERACY



Mā te mōhio ka ora:
mā te ora ka mōhio

Through learning there is life:
through life there is learning!

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LEARNING PROGRESSIONS

FOR ADULT NUMERACY

Foreword

E ngā iwi, e ngā reo, e ngā mana, e rau rangatira mā.

Tēnā koutou, tēnā koutou, tēnā tatou katoa.

We are delighted to release this latest version of the learning progressions, knowing that many people involved with adult literacy and numeracy in New Zealand are keen to put it to use.

Thank you so much to the many people who have contributed to the production of this document, through the preparation of material and by providing advice and suggestions for improvement.

As we continue our effort to ensure that every New Zealander has the crucial literacy and numeracy skills they need for living and learning, the progressions offer a robust framework for other tools and resources, a focus for continuing to develop high quality teaching and learning, and a common language for use in the many settings where literacy and numeracy are developed.

Supporting adults to develop these skills is not as easy as ABC. This sector will always need evidence-based research, informed managers and dedicated tutors who are committed to the challenge of improving their teaching. These progressions, with their accent on strengthening learners' expertise, are key tools in our "kete" to help us rise to the task.

Mā te mōhio ka ora: mā te ora ka mōhio

Through learning there is life: through life there is learning!

Janice Shiner
Chief Executive
Tertiary Education Commission

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Introduction

To work and participate effectively in a modern knowledge society, New Zealand adults require a certain level of numeracy expertise. Adults who have not yet developed this level of expertise will find it difficult to fully participate. If they and their teachers understand how expertise develops, and if they have a clear picture of the steps they can take, they will be able to make progress in learning to solve number problems, reason statistically and measure and interpret shape and space.[1]

The learning progressions set out on in this book provide a framework that shows what adult learners know and can do at successive points as they develop their expertise in numeracy learning. This framework can be used as guide when identifying the next steps for adult learners. Each progression covers a particular aspect of learning.

The progressions can be used in many different adult learning settings. They describe what is learned in the order that it is usually learned. They can be used:

- to gain a basic picture of an adult learner's current skills, strategies and knowledge in numeracy
- to identify the numeracy-related demands of specific workplace, community or personal tasks, problems and texts, and
- as a source of sequences for teaching and learning programmes and of suggestions to use in designing such programmes.

The progressions are neither a curriculum nor a teaching and learning programme. They are not, as they stand, an assessment tool, and they are not a set of teaching and learning activities. Rather, teachers and managers of foundation-level adult numeracy learners are invited to use the progressions as a basis for developing or adapting their own curricula, programmes, assessment tools and teaching and learning activities.

The professional development resources that accompany this booklet will suggest some ways to develop materials based on the progressions for many different purposes. They also include some models for task and problem analysis and diagnostic assessment, as well as a range of teaching and learning activities.

The learning progressions do not describe all of the knowledge and skills needed to meet any specific achievement standards or Unit Standards of the National Qualifications Framework (NQF).[2] However, the progressions do have a natural link to the national assessment system. The highest step in each progression describes the knowledge and skills that underpin the numeracy competencies demonstrated by students with level 2 or 3 NQF qualifications. Adult learners who acquire all the knowledge and skills described in the relevant progressions will have a sound foundation to build on if they decide to study for achievement standards or Unit Standards at level 3, 4 or 5 of the NQF.

The research that informed the development of the learning progressions is described in a companion booklet, *Learning Progressions for Adult Literacy and Numeracy: Background Information.*

1 Adults also need basic language and literacy knowledge, skills and strategies. The language and literacy progressions are presented in a companion booklet, *Learning Progressions for Adult Literacy.*

2 Students within the school sector work towards these standards in order to get the National Certificate of Educational Achievement (NCEA), and many adult learners also work towards them, for example, in adult learning courses in institutes of technology and polytechnics.

The structure of the progressions

The strands

A strand of thread is made up of many individual fibres. In the same way, each strand of the learning progressions is made up of several progressions, which together describe the development of expertise within the strand. The learning progressions for numeracy are organised in the following three strands:

- Make Sense of Number to Solve Problems
- Reason Statistically
- Measure and Interpret Shape and Space.

The strands are interconnected. For example, learners need an understanding of place value (covered in the strand Make Sense of Number to Solve Problems) in order to convert between units of measurement (in the strand Measure and Interpret Shape and Space).

The progressions

The term progression is used to describe a set of steps along a continuum, each step representing a significant learning stage as learners build their expertise. Each progression highlights a particular area of learning within a strand. The progressions are intended to illustrate a typical learning pathway.[3] The titles of the progressions are listed on page 7.

A progression implies a continuous, sequential movement towards expertise rather than a series of separate tasks to be mastered in order to "move up". For this reason, individual steps within a progression are distinguished from one another in this book by referring to their place in the sequence (for example, "the second step in the Additive Strategies progression") rather than by using numbers, stages or levels. The learning progressions reflect the way all learners continually build on and extend their existing knowledge and skills.

The progressions are also interconnected. For example, the learners need to be able to sort and record data (in the progression Preparing Data for Analysis) in order to describe and interpret that data (in the progression Analysing Data for Interpretation).

The steps

Development within any one progression is not evenly spaced, and some of the shifts in development involve more learning than others. The amount of learning needed will also depend on the learner. Adults do not all learn in the same way. Some need to spend more time learning certain skills, or consolidating the learning, than others.

On the pages that show each progression, the steps towards expertise in that progression are represented by pikopiko that have increasing numbers of fronds. The initial learning step is represented by a single koru, the next step by a pikopiko with two fronds and so on. The final step is represented in most cases by a pikopiko with six fronds.

The koru (in its mature forms, the pikopiko) was chosen as the symbol for the steps in each progression because it is a familiar and valued image for New Zealanders and because its natural and gradually unfolding growth pattern could be seen to reflect the process of successful learning, or ako. As fronds mature, new fronds begin to grow, nourished and sheltered by the work of the existing fronds, the plant's root system and a favourable environment. Pikopiko is an indigenous food picked directly from ngahere (the forest) that can give and sustain life. In the same way, ako can give and sustain intellectual and spiritual life.

3 Although no adult learner is ever completely typical, there are typical patterns of progress common to the majority of adult learners.

The steps vary in size and quantity from one progression to another. This variation is because the writers have tried to show steps at parallel stages of a typical learner's development across all the progressions. The steps do not, however, all involve the same amount of learning, and the development of skills, strategies and knowledge does not always occur in evenly sized or spaced steps.

In the Make Sense of Number to Solve Problems strand, for example, only two progressions (the Additive Strategies progression and the Multiplicative Strategies progression) have six separate steps. In this and other strands, there are some progressions that have double steps (the movement in the progression is shown over two steps) because the learning described by the bullet points takes time to develop, consolidate and practise. This is considered to be the equivalent of two steps in a single progression.

A different kind of variation occurs in places where learning in one progression depends on prior learning in another. For example, numeracy learners cannot begin learning about place value, which involves counting in tens, until they have learned to count in ones, and this learning occurs at the first step of the Additive Strategies progression. Because of this, there is a gap at the first step in the progression for Place Value.

Numeracy progressions

Make Sense of Number to Solve Problems

Additive Strategies progression

Multiplicative Strategies progression

Proportional Reasoning Strategies progression

Number Sequence progression

Place Value progression

Number Facts progression

Reason Statistically

Preparing Data for Analysis progression

Analysing Data for Interpretation progression

Interpreting Data to Predict and Conclude progression

Probability progression

Measure and Interpret Shape and Space

Shapes and Transformations progression

Location progression

Measurement progression

Make Sense of Number to Solve Problems
Additive Strategies progression

Adults use counting and partitioning strategies to add and subtract. These strategies make it easier to solve problems that involve numbers and to develop numeracy expertise. See page 23 for information about number strategies.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:

• solve addition and subtraction problems by counting all of the objects.	Learners solve simple addition and subtraction problems by counting all of the objects. Typically, a learner will use fingers, counters or other objects. For example, a learner may add 8 + 7 by starting from 1, counting 8 objects, then continuing to count 7 more objects to reach 15.
• solve addition and subtraction problems by counting on or counting back, using ones and tens.	Learners use "in the head" (mental) strategies. They can count on (for addition) or back (for subtraction) from the first number given. They do not rely on fingers or other objects, and they can count in ones and in tens and in combinations of ones and tens. – **Counting in ones**: A learner may solve 8 + 5 by starting at 8, then mentally counting 5 more, by ones, to reach 13. – **Counting in tens and ones**: A learner may solve 46 - 23 by starting at 46, then mentally counting 20 back, by tens, to reach 26 and then counting 3 back, by ones, to reach 23.
• solve two-digit by one-digit addition and subtraction problems mentally, using partitioning strategies.	Learners use mental strategies that require them to partition numbers (that is, to split numbers into parts). Partitioning strategies include the following: – **Deriving from known facts**: Learners derive unknown information from a known fact. A learner may solve 26 + 6 by using what they know (6 + 6 = 12), then adding 20 + 12 to reach 32. – **Making tens**: Learners partition numbers in order to be able to work from the nearest ten. A learner may solve 16 + 7 by splitting the 7 into 4 and 3, adding 16 + 4 to reach 20, then adding the 3 to reach 23. – **Using tidy numbers with compensation**: Learners round a number to the nearest ten or hundred, then compensate for what has been added or subtracted. Learners know that, in the addition process, if they add something to a number, they must take it away again at the end (26 + 9 can be solved as 26 + 10 - 1 = 35) and that, in subtraction, if they take something away from a number, they must add it back on at the end (53 - 9 can be solved as 53 - 10 + 1 = 44).
• solve multi-digit addition and subtraction problems, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	Learners use partitioning strategies to solve more complex addition or subtraction problems. Partitioning strategies include the following: – **Deriving from known facts**: Learners derive unknown information from a known fact, for example, solving 25 + 26 by first adding 25 + 25 to get 50, then adding 1 more to get 51. – **Making tens**: Learners partition numbers in order to be able to work from the nearest ten. For example, a learner can solve 45 + 37 by splitting the 37 into 5 and 32, then adding 45 + 5 to get 50 and then adding 50 + 32 to get 82. Or the learner can solve 64 - 37 by splitting the 37 into 4 and 33, subtracting 64 - 4 to get 60 and then subtracting 33 from 60 to get 27. – **Place value partitioning**: The learner breaks the numbers into ones, tens and hundreds, adds numbers of the same place value together and then combines these numbers. For example: 23 + 34 can be solved as (20 + 30) + (3 + 4) = 50 + 7 = 57. 657 - 234 can be solved as (600 - 200) + (50 - 30) + (7 - 4) = 400 + 20 + 3 = 423. – **Using tidy numbers with compensation**: The learner rounds a number to the nearest ten or hundred, then compensates for what has been added or subtracted. For example, 46 + 19 can be solved as 46 + 20 - 1 = 65. Alternatively, learners may use a calculator or written algorithm to solve a problem. If so, they are able to justify the solution by demonstrating or explaining why it is reasonable. – **Standard algorithm explanation**: $^{1}37$ + 28 65 65 is reasonable because 37 + 28 is a bit less than 70 (40 + 30). $^{6}\not{7}\,^{14}\not{4}$ - 3 8 3 6 36 is reasonable because 74 - 38 is a bit more than 34 (74 - 40).
• solve addition and subtraction problems involving decimals and integers, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	Learners use partitioning strategies to solve addition and subtraction problems involving decimals and integers. Partitioning strategies include the following: – **Using tidy numbers with compensation**: For example, a learner solves 3.2 + 1.95 as 3.2 + 2 - 0.05 = 5.2 - 0.05 = 5.15. – **Place value partitioning**: Learners partition numbers by place value. For example, a learner can solve 6.03 - 5.8 by subtracting 6.03 - 5 to get 1.03, then subtracting 1.03 - 0.8 to get 0.23. – **Using reversibility**: Learners change a subtraction problem into an addition problem. For example, 6.03 - 5.8 becomes 5.8 + ? = 6.03, and -15 + 64 becomes 64 - 15. Alternatively, learners may use a calculator or written algorithm to solve a problem. If so, they are able to justify the solution by demonstrating or explaining why it is reasonable.
• solve addition and subtraction problems involving fractions, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	Learners use partitioning strategies to solve addition and subtraction problems involving fractions. Partitioning strategies include the following: – **Using equivalent fractions**: Learners use knowledge of equivalent fractions to solve addition and subtraction problems. For example, $\frac{5}{6} + \frac{7}{4} = \frac{10}{12} + \frac{21}{12} = \frac{31}{12}$. Alternatively, learners may use a calculator or written algorithm to solve a problem. If so, they are able to justify the solution by demonstrating or explaining why it is reasonable.

Make Sense of Number to Solve Problems
Multiplicative Strategies progression

Adults use strategies to multiply and divide. These strategies include counting and partitioning strategies (for example, counting all the objects) to solve multiplicative problems. Adults can explain why the solutions they reach using an algorithm or calculator are reasonable. See page 23 for information about number strategies.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:

• solve multiplication problems by counting all the objects.	Learners solve simple multiplication problems by counting all the objects. Typically, learners will use fingers, counters or other objects. For example, a learner may solve 2 x 3 by counting two groups of three, each with three ones (1, 2, 3; 4, 5, 6).
• solve multiplication problems by skip-counting, often in conjunction with one-to-one counting and often keeping track of the repeated counts by using materials (for example, fingers) or mental images.	Learners solve simple multiplication problems by skip-counting. For example, a learner may solve 4 x 5 by skip-counting in fives (5, 10, 15, 20).
• solve single-digit multiplication and division problems mentally, using known multiplication facts and repeated addition.	Learners solve single-digit multiplication and division problems, using repeated addition or deriving unknown information from known multiplication and division facts. Examples can include the following: – **Repeated addition**: A learner may solve 4 x 6 by adding 6 + 6 + 6 + 6 to reach 24. – **Deriving from known facts**: A learner may solve 72 ÷ 8 = 9 by recalling that 8 x 8 = 64 and then adding one more 8 to get 72. Alternatively, a learner may solve this by recalling that 8 x 10 = 80 and then subtracting one 8 to get 72.
• solve multiplication and division problems with single-digit multipliers or divisors mentally, using partitioning strategies and deriving from known multiplication facts.	Learners use mental strategies to solve multiplication and division problems that have single-digit multipliers or divisors. Learners use mental strategies that are based on derivations from known multiplication or division facts. Partitioning strategies include the following: – **Deriving from known facts**: Learners partition mentally to allow the use of known number facts first, for example, 10 x 13 = 130 so 9 x 13 = 130 – 13 = 117. – **Place value partitioning**: Learners break numbers into tens, for example, 14 x 5 can be solved as (10 + 4) x 5. 10 x 5 = 50 and 4 x 5 = 20, so the solution is 50 + 20 = 70. – **Using reversibility**: Learners change a division problem into a multiplication one and use known facts, for example, 72 ÷ 4 can be solved as 4 x ? = 72. The learner knows that 4 x 20 = 80, and 8 ÷ 4 is 2, so 4 x 18 = 72. (Note that this problem involves deriving from known facts as well as using reversibility.)
• solve multiplication or division problems with multi-digit whole numbers, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	Learners solve multi-digit multiplication and division problems with whole numbers by using partitioning strategies. Partitioning strategies include the following: – **Deriving from known facts**: Learners use known facts to reach a solution. 4 x 25 = 100, so 8 x 25 = 100 + 100 = 200. 72 ÷ 4 can be solved as 72 ÷ 2 = 36, 36 ÷ 2 = 18 (because dividing by 4 is the same as dividing by 2 twice). – **Using equivalent expressions**: 360 ÷ 5 can be solved as 720 ÷ 10 = 72. 81 ÷ 3 can be solved as 9 x (9 ÷ 3) = 9 x 3 = 27. – **Place value partitioning**: Learners partition numbers by using knowledge of place value. For example, 24 x 36 can be solved as 20 x 36 + 4 x 30 + 4 x 6 = 720 + 120 + 24 = 864. Alternatively, learners may use a calculator or written algorithm to solve a problem. If so, they can justify the solution by demonstrating or explaining why it is reasonable. – **Calculator (with explanation)**: Learners use a calculator to find the answer and are able to explain why the answer obtained is reasonable. For example: 45 x 23 = 1,035 (using a calculator). "1,035 is a reasonable answer because I know that 45 x 20 is 900."
• solve multiplication or division problems with decimals, fractions and percentages, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	Learners solve multiplication or division problems with decimals, fractions and percentages, using partitioning strategies. – **Converting between fractions and percentages** 25% off $56 is $42. The learner knows that 25% is $^1/_4$ and $^1/_4$ of 56 is 14. 56 – 14 = 42. Alternatively, learners may use a calculator or written algorithm to solve a problem. If so, they can justify the solution by demonstrating or explaining why it is reasonable. – **Standard algorithm (with reasonableness)** $^2/_3$ x $^3/_4$ = $^6/_{12}$ (by multiplying the 2 x 3 and the 3 x 4). "This is reasonable because I know that $^2/_3$ of 1 is $^2/_3$ and $^2/_3$ is a little more than a half ($^6/_{12}$)."

Make Sense of Number to Solve Problems
Proportional Reasoning Strategies progression

Adults use proportional reasoning strategies. These strategies enable adults to solve problems that involve counting, equal sharing, multiplication and division, fractions, decimals, percentages, proportions, ratios and rates. See page 23 for information about number strategies.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:	
	There is a gap at the first step of this progression because learners need to be able to count all objects before they can use the strategy of equal sharing.
• find a fraction of a set by using equal sharing.	Learners use the strategy of equal sharing to find fractions of a set.
	There is a gap at the third step of this progression because learners need to know single-digit multiplication and division facts before they can use them to find fractions of whole numbers.
• use known multiplication and division facts to find fractions of a whole number.	Learners use multiplication and division facts to find unit fractions of whole numbers. For example: – $^{1}/_{3}$ of 24 = 8 because 24 ÷ 3 = 8, which is $^{1}/_{3}$. – $^{1}/_{5}$ of 35 = 7 because 35 ÷ 5 = 7.
• use multiplication and division strategies to solve problems that involve simple equivalent fractions and simple conversions between fractions, decimals and percentages.	Learners use strategies to solve problems that involve simple equivalent fractions and simple conversions between fractions, decimals and percentages. For example, a learner knows that: – they can find $^{3}/_{12}$ of a number by dividing by 4 because $^{3}/_{12}$ = $^{1}/_{4}$ – $^{1}/_{4}$ is the same as 0.25 is the same as 25% – $^{1}/_{2}$ of $^{1}/_{4}$ is $^{1}/_{8}$. Learners solve problems by deriving from known fractions, decimals or percentages. For example, a learner: – finds 20% of 80 by knowing that 10% is 8 and doubling to get 16 – finds $^{1}/_{5}$ of 70 by knowing that $^{1}/_{10}$ is 7 and doubling to get to 14 – finds 0.8 of 40 by knowing that $^{1}/_{10}$ (0.1) is 4 and 4 x 8 is 32. Learners use calculators to solve problems involving fractions, decimals and percentages and are able to explain the reasonableness of the answer. For example, "I will divide by 100 on my calculator to find 1% and then multiply by 17 to get 17%" or, alternatively, "17% of 80: 10% is 8 and 20% is 16, so my answer (13.6) is reasonable."
• use multiplication and division strategies to solve problems that involve proportions, ratios and rates.	Learners use strategies to solve problems that involve proportions, ratios and rates. – **Using common factors** For example, 15:18:21 is the same as 10: ? : ? . The common factor is 3. 10 is $^{2}/_{3}$ of 15. 12 is $^{2}/_{3}$ of 18. 14 is $^{2}/_{3}$ of 21. – **Converting fractions to percentages** For example, 70 is what percentage of 42? $^{70}/_{42}$ = 1 $^{28}/_{42}$ = 1 $^{4}/_{6}$ = 1 $^{2}/_{3}$ = 166.66%. – **Determining the "unit"** For example, a learner solves this problem: The tank is $^{4}/_{5}$ full of water. Andrew will use $^{2}/_{3}$ of that water to water his plants (200 litres). How much water does the tank hold when full? That is, $^{2}/_{3}$ of $^{4}/_{5}$ of ? =200; $^{1}/_{3}$ = 100 litres; $^{1}/_{3}$ of $^{4}/_{5}$ is $^{4}/_{15}$. If $^{4}/_{15}$ = 100, then $^{1}/_{15}$ is 25. 15 x 25 = 375 litres. $^{1}/_{3}$ of the water would be 100 litres. $^{1}/_{3}$ of $^{4}/_{5}$ is $^{4}/_{15}$. If $^{4}/_{15}$ of the tank is 100 litres, then $^{1}/_{15}$ is 25 litres, and 15 times 25 equals 375 litres. Learners can calculate rates by comparing two quantities or measurements that have different units, for example: – If it takes 2 hours to travel 130 kilometres, the average speed for the journey (the total distance travelled in one time unit) is 130 kilometres per 2 hours = 65 kilometres per hour. Learners can calculate rates to make comparisons, for example: – If 3 kilograms of potatoes costs $3.60, the price per kilogram is $3.60 per 3 kilograms = $1.20 per kilogram. This charge per kilogram is a rate. (At supermarkets, such rates are often expressed as cents per 100 grams so that the purchaser can compare prices of similar items.)

Make Sense of Number to Solve Problems
Number Sequence progression

Adults are able to use knowledge of number strategies (including counting strategies) to solve problems that involve number sequence. See page 23 for information about number strategies.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL KNOW:

• the sequence of numbers, forwards and backwards, to at least 20.	Learners can count forwards and backwards, in order, to at least 20. They know where numbers come in sequence, for example, a learner knows that the number after 16 is 17 and that the number before 16 is 15.
• the sequence of numbers, forwards and backwards, to at least 100 • how to skip-count in twos, fives and tens to 100.	Learners can count backwards and forwards to at least 100. They know where numbers come in sequence, for example, a learner knows that the number 38 comes after 37 and before 39. Learners can count forwards in twos, fives and tens (skip-counting) to 100.
• the sequence of numbers, forwards and backwards, to at least 1,000 • the number that is 1, 10 and 100 before or after a given number in the range 0–1,000 • how to skip-count in twos, threes, fives and tens to 1,000 • how to order fractions with like denominators.	Learners know the sequence of numbers, forwards and backwards, to at least 1,000. They know where numbers come in sequence, using tens and hundreds as guides. For example, a learner knows that 473 comes before 528 because there are five hundreds in 528 and only four in 473. They know that 382 is ten more than 372 and ten less than 392. Learners can skip-count in twos, threes, fives and tens, for example, 113, 116, 119, 121. Learners know that the denominator indicates the number of equal parts a whole is divided into and that the numerator is the number of those parts. Learners can order fractions that have the same denominator (such as $^{1}/_{5}$, $^{2}/_{5}$, $^{3}/_{5}$, … $^{31}/_{5}$).
• the sequence of numbers, forwards and backwards, by ones, tens, hundreds and thousands, to a million • how to give the number 1, 10, 100 or 1,000 before or after a given number in the range 0–1,000,000 • the sequence of decimal numbers in tenths and hundredths • how to order unit fractions.	Learners know the sequence of numbers and can count using ones, tens, hundreds and thousands, up to a million. For example, 10,000, 20,000, 30,000 …. Learners know the numbers that come before and after any given number, using ones, tens and thousands. For example, a learner knows that 3,904 comes after 3,812 and before 4,132 and that 242,500 comes after 141,400 and before 343,200. They can place numbers up to 1,000,000 in order. Learners know the sequence of decimal numbers, for example, that 5.304 comes after 5.204 and before 5.404. Learners can order unit fractions (fractions that have 1 as the numerator, such as $^{1}/_{5}$, $^{1}/_{4}$, $^{1}/_{3}$, $^{1}/_{2}$).
• the sequences of integers, fractions, decimals and percentages, forwards and backwards, from any given number.	Learners know the sequences of integers, decimals, fractions and percentages and can order these from any given number. For example: – 1.376, 1.377, 1.378 – 3.387, 3.4, 3.418, 3.42 – $^{3}/_{8}$, $^{1}/_{2}$, $^{2}/_{3}$, 3.4 – 10%, 0.45, 0.759, 80%.

Make Sense of Number to Solve Problems
Place Value progression

Adults understand place value. They use their knowledge of number strategies (counting and partitioning) as they solve problems that involve place value. See page 23 for information about number strategies.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL KNOW:		
		There is a gap at the first step of this progression because learners need to be able to count all objects before they can learn to count in tens.
	• 10 as a counting unit, the tens in numbers to 100 and the place values of digits in whole numbers up to 100.	Learners know how many tens there are in numbers to 100 and can partition numbers up to 100 into tens and ones, for example: – 6 tens are 60 – 34 = 30 + 4.
	• the tens and hundreds in numbers to 1,000 and the place values of digits in whole numbers up to 1,000.	Learners know how many tens and hundreds there are in numbers up to 1,000 and can partition numbers up to 1,000 into hundreds, tens and ones, for example: – there are 64 tens in 645 (and 5 ones remaining) – 561 rounded to the nearest 10 is 560 – 456 = 400 + 50 + 6.
	• how many tens, hundreds and thousands there are in any whole number • that 10 tenths make one whole.	Learners know how many tens, hundreds and thousands there are in any whole number and know that 10 tenths make one whole. For example: – there are 4,560 thousands in 4.56 million – there are 500 hundreds in 50,000 – there are 20 tenths in 2.
	• how many tenths, hundredths and thousandths are in any number, including decimal numbers • how to convert percentages to decimals and vice versa • what happens when a whole number or decimal is multiplied or divided by a power of 10.	Learners can order and convert between tenths, hundredths and thousandths. For example, a learner knows that: – there are 3,420 tenths in 342 – 137.5% is the same as 1.375 – 6.49 rounded to the nearest tenth is 6.5 – 4.5 x 100 = 450 – 78.3 ÷ 100 = 0.783.

Make Sense of Number to Solve Problems
Number Facts progression

Adults know number facts. These include addition, subtraction, multiplication and division facts as well as fraction, decimal and percentage facts. Adults use their knowledge of number strategies and apply these facts as they solve problems. See page 23 for information about number strategies.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL KNOW:	
• addition facts with sums of 5 or 10 and the decade facts.	Learners know addition facts with sums of 5, for example, 1 + 4 = 5, 2 + 3 = 5, 4 + 1 = 5. Learners know addition facts with sums of 10, for example, 1 + 9 = 10, 2 + 8 = 10, 7 + 3 = 10. Learners know the "decade facts" (that is, how to add any single digit to any multiple of 10), for example, 10 + 4 = 14, 40 + 7 = 47.
• basic addition and subtraction facts up to 10 + 10.	Learners know the basic addition and subtraction facts up to 10 + 10, for example: – 3 + 4 = 7, 4 + 3 = 7 – 7 - 3 = 4, 7 - 4 = 3 – 9 + 9 = 18, 18 - 9 = 9.
• basic multiplication and division facts up to 10 x 10.	Learners know the basic multiplication and division facts up to 10 x 10. For example, a learner knows that: – the 5 times table is used to multiply: 1 x 5 = 5, 2 x 5 = 10 – the 5 times table is used to divide: 50 ÷ 5 = 10, 45 ÷ 5 = 9 – 6 x 4 = 24, 3 x 7 = 21, 8 x 4 = 32 – 56 ÷ 7 = 8, 42 ÷ 6 = 7.
• basic multiplication facts with tens, hundreds and thousands • fraction and decimal groupings that make 1.	Learners know multiplication and division facts with tens, hundreds and thousands, for example: – 20 x 300 = 6,000 – 400 x 800 = 32,000 – 5,000 ÷ 10 = 500 – 10 million ÷ 200 = 50,000. Learners know the fraction and decimal groupings that make 1, for example: – $^3/_4 + ^1/_4 = 1$ – $^2/_7 + ^5/_7 = 1$ – 0.6 + 0.4 = 1.
• common factors of numbers up to 100 • fraction, decimal and percentage conversions for halves, thirds, quarters, fifths and tenths • the convention for exponents.	Learners know common factors, for example: – that the common factors of 48 and 64 are {1, 2, 4, 8, 16}. Learners know the fraction, decimal and percentage conversions for halves, thirds, quarters, fifths and tenths. For example, a learner knows that: $^3/_4$ = 0.75 = 75%. Learners understand and can use the convention for exponents. For example, a learner knows that: 2^4 (2 to the power of 4) = 2 x 2 x 2 x 2 = 16.

Reason Statistically
Preparing Data for Analysis progression

The ability to work with data is an important part of using and understanding statistics in everyday life. See page 23 for information about the development of the ability to reason statistically.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:	
• sort objects according to their attributes, organise data about the objects and represent data, using concrete objects or pictures.	Learners can: – identify potentially relevant attributes of objects in a collection – use a specific attribute as a basis for sorting a collection of objects – draw a diagram to represent the objects that make up a collection, in their different categories.
• sort and organise category data and represent it, using tables, pictographs and bar graphs.	Learners can: – identify and describe categories to be used as the basis for gathering data – count data and sort it into categories – order data by category or frequency – represent category data, using tables, pictographs or bar graphs.
• sort, organise and represent data, using tables and graphs such as line plots, bar graphs and line graphs • recognise the differences involved in representing category and numeric data.	Learners can: – read and discuss numeric data presented in a simple table format – distinguish between data based on a single variable and data based on two variables – calculate the median and range of a data set – represent data in one or two variables in the form of tables, line plots, bar graphs and stem-and-leaf plots.
• sort, organise, clean and represent multi-variate data, making appropriate use of histograms, stem-and-leaf plots, box plots (box-and-whisker diagrams) and scatter plots • graph time-series data.	Learners can: – clean a data set by eliminating irrelevant data or obviously erroneous data – create a variety of graphical representations and determine which best shows the key features of the data.

Reason Statistically
Analysing Data for Interpretation progression

The ability to analyse data is an important part of using and understanding statistics in everyday life. See page 23 for information about the development of the ability to reason statistically.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:	
• describe parts of the data and the set of data as a whole to determine what the data show.	Learners can: – use visual representations of data to make comparisons between different groups of objects (for example, relative size).
• describe the general features of a data set.	Learners can: – use representations of category data to make statements about the relative importance of the different categories – identify and describe clusters of data and any unusual features.
• describe the shape and important features of a sample data set (considering especially median and range) • compare two or more samples.	Learners can: – compare median and range for several sets of sample data and, in this way, roughly estimate an overall median.
• find, use and interpret measures of centre and spread, including mean and interquartile range.	Learners can: – explain how the mean and median relate to the shape of a data set – choose an appropriate measure of centre (median or mean) – recognise weaknesses of the range and explain why *interquartile* range may be a better measure of spread – examine a box plot and discuss the important features of the data set it represents.

Reason Statistically
Interpreting Data to Predict and Conclude progression

The ability to interpret data is an important part of using and understanding statistics in everyday life. See page 23 for information about the development of the ability to reason statistically.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:

	There are gaps at the first steps of this progression because learners need to be able to sort, organise and describe data before they can interpret data. They need to be able to prepare and analyse data in order to interpret it.
• make sensible statements based on the general features of a data set.	Learners can: – talk about the shape of a data display in terms of symmetry, clusters and spread – use terms such as *most common, least common, unusual gaps* or *unusual peaks* to describe features of the data set or display – discuss or explain any differences between what their data set/display shows and what they had expected to find or see – explain how data can be used to provide greater certainty and more accurate predictions.
• draw conclusions and make predictions, based on evidence from the data.	Learners can: – make summary statements about data, referring to features such as clusters, range, median and subsets within the larger data set – discuss trends in the data and make evidence-based predictions – discuss possible reasons for variation between one data set and other similar sets of data – discuss data-based summaries and supporting graphic displays in terms of certainty – make predictions that extend beyond basic data interpretation to include elements of uncertainty, acknowledging the role of variation.
• use observations based on samples to make conjectures about the populations from which the samples were taken.	Learners can: – discuss the features of the normal curve and explain how it models the spread of a typical random variable (for example, height or weight) – use two or more sample data sets to make generalisations about a larger population, estimating, for example, the mean of a particular variable, its spread or the likelihood of a particular event occurring – compare data sets and displays as a means of learning to determine the reasonableness of their own generalisations – identify and discuss sources of potential bias and distortion (in sampling methods and in statistical summaries and graphic displays) – acknowledge sources of uncertainty, based on an awareness of variability.

Reason Statistically
Probability progression

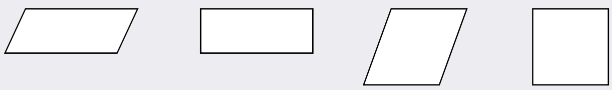
Many everyday applications of statistics require an understanding of the nature of probability and how mathematical models and relative frequency can be used to describe the likelihood of particular outcomes (events). See page 23 for information about the development of the ability to reason statistically.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:	
• identify all possible outcomes in situations involving simple (single-stage) chance • use words to describe the likelihood of particular outcomes (events).	Learners identify all possible outcomes in situations such as tossing a coin and rolling a dice. Learners use words to position events on a continuum that goes from *impossible* to *certain.*
• use fractions to express the probability of events • recognise uncertainty in simple (single-stage) chance situations.	Learners use fractions to express probabilities in simple chance situations. For example, if a coin is tossed, the probability of it landing as a head is $^1/_2$. Learners know that, in such situations, the outcome is not influenced by earlier outcomes. If a coin is tossed three times, and three heads are obtained, this makes it no more or less likely that the result of the next toss will be a head.
• use relative frequency to provide an estimate of the probability of an event • use fractions, ratios and percentages to express probabilities • compare the results of trials or observations with expectations based on models.	Learners understand that they can estimate the likelihood of an event by doing repeated trials or observations and then comparing the number of 'successes' with the number of trials. For example, the probability that a goal shoot will get her next shot in the net can be estimated by comparing successes with attempts over a number of games. For simple chance events, learners can interpret probabilities expressed as fractions, percentages and ratios (odds). Learners understand that, while they can't determine which of the various possible outcomes will occur next, they may be able to determine mathematically the probabilities of each outcome. For example, the probability of drawing an ace from a standard pack of 52 playing cards is $^4/_{52} = {}^1/_{13}$.
• determine the probabilities in simple multi-stage probability situations • apply the law of large numbers to probability situations.	In simple, multi-stage chance situations (where the events are independent), learners can determine probabilities by using, for example, systematic lists or two-way arrays (tables). Learners can recognise complementary and mutually exclusive events. For example, a learner knows that when two dice are rolled, 'double' and 'not a double' are both complementary (one or other must occur) and mutually exclusive (they can't both occur) events. Learners know there are situations in which probabilities cannot be determined theoretically. In such cases, the relative frequency of an event can be used to estimate its probability. Learners know that, in determining the relative frequency of an event, the greater the number of trials, the more accurate the estimate (this is the law of large numbers). For example, the accuracy of a particular weather forecaster can't be determined from a single forecast.
• determine the probabilities in more complex multi-stage chance situations • apply the notion of 'expected value' to probability situations.	Learners can: – understand that the probability of an event can sometimes depend on previous outcomes (such events are said to be dependent), and a learner can apply this understanding to more complex probability situations – determine probabilities in multi-stage chance situations. For example, the probability of a first-division Lotto win is $^6/_{40}$ x $^5/_{39}$ x $^4/_{38}$ x $^3/_{37}$ x $^2/_{36}$ x $^1/_{35}$ (about 1 in 4 million). Situations can be modelled using such means as systematic lists or tree diagrams – apply the notion of 'expected value' to probability situations such as games (with pay-offs) and insurance premiums. For example, if the 'house advantage' on a gaming machine is 10 percent, over the long run, punters will get back only 90 percent of what they paid in.

Measure and Interpret Shape and Space
Shapes and Transformations progression

Adults are able to sort, describe, define and create representations of shapes and the ways in which they can be transformed in space. They use strategies to apply their knowledge of shapes to tasks in their everyday lives. See page 24 for information about the development of the ability to measure and interpret shape and space.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:

• sort and describe objects by their shape attributes.	Learners can: – sort objects by shape attributes into groupings, such as cylinders and boxes – describe the shape attributes of objects, for example, a box has 6 rectangles as faces.
• identify and describe plane shapes in objects • record the result of transformations (flips, turns and slides) on plane shapes.	Learners can: – identify plane shapes, including triangles, rectangles and circles in everyday objects – record the result of flipping, sliding or turning plane shapes.
• create mental images of plane shapes • recognise and represent plane shapes in objects from different perspectives • predict and communicate the results of transformation (flips, turns, slides and/or scaling) on plane shapes.	Learners can: – identify and sketch (from observation involving different points of view and perspectives) plane shapes – predict the result of flipping, sliding, turning and/or scaling plane shapes.
• define plane shapes and prisms by their spatial features • create and describe mental images of prisms, including cylinders • make two-dimensional representations of prisms (and vice versa) • describe the transformations (flips, turns, slides and/or scaling) that are used to map one object onto another.	Learners define plane shapes and prisms by their spatial features. For example, a learner can make, state, draw and describe: – a triangle as a plane shape that has 3 straight sides and 3 corners (vertices) – a rectangle as a plane shape that has 4 straight sides (quadrilateral) and 4 right-angled vertices – a parallelogram as a quadrilateral with parallel opposite sides.
• define classes of plane shapes by their geometric properties and classes of solid shapes by their surfaces • use spatial visualisation to solve problems that involve surface area and volumes of prisms • describe sizes, positions and orientations of shapes under transformation (flips, turns, slides and/or scaling).	Learners can define classes of plane shapes by their geometrical properties and classes of solid shapes (such as cuboids and spheres) by their surfaces. For example, a learner can describe the relationships among types of parallelograms, such as a rhombus, square and rectangle. Parallelogram Rectangle Rhombus Square 
• visualise three-dimensional objects and spaces from different perspectives and analyse their cross-sections • examine the congruence, similarity and line or rotational symmetry of objects, using transformations.	Learners can visualise and describe three-dimensional objects and spaces from different perspectives and know the congruence, similarity and line or rotational symmetry of objects. For example, a learner knows that: – all squares have rotational symmetry of order 4 and therefore all squares have 4 right-angled vertices and 4 congruent sides with parallel opposite sides – all squares have 4 congruent sides with parallel opposite sides, 2 axes of symmetry that are perpendicular bisectors of adjacent sides of the square and 2 axes of symmetry that bisect the vertices to make angles of 45 degrees.

Measure and Interpret Shape and Space
Location progression

Adults use spatial knowledge in relation to location. They apply their knowledge of shapes and location of shapes in space to tasks in their everyday lives. See page 24 for information about the development of the ability to measure and interpret shape and space.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:	
• describe, name and interpret relative positions in space.	Learners can use everyday language to describe the location of objects in physical space or represented in drawings or photographs. Such language includes: *under, above, on top of, below, beside, to the left of, to the right of, nearby, behind* and *in front of.*
• give and follow instructions for movement that involve distance and directions.	Learners can devise instructions for a street route for someone else to follow and draw the street route described in a set of instructions they are given, for example, "Turn right out of the gate, take the next left, and then take the third street on your right. Proceed for 300 metres until you see a yellow fence."
• use grid co-ordinate systems to specify locations and to describe routes.	Learners can find locations specified by a map index and can devise the shortest practicable route from one specified location to another.
• communicate and interpret locations, directions and distances, using bearings, grid references and scales.	Learners can: – devise and justify their choice of the shortest sensible route, using bearings, grid references and terrain contours, for example, during an outdoor orienteering exercise – use the scale on a map and their knowledge of sensible average travelling speeds to predict an estimated time of arrival (ETA), for example, for a car trip or for someone on a tramping trip.

Measure and Interpret Shape and Space
Measurement progression

Adults use strategies to measure. They can compare, order and measure objects, selecting appropriate units, tools, estimates and formulas for tasks in their everyday lives. See page 24 for information about the development of the ability to measure and interpret shape and space.

AS THEY DEVELOP THEIR EXPERTISE, MOST ADULTS WILL BE ABLE TO:	
• compare and order objects directly, using attributes of length, area, volume and capacity, weight, angle, temperature and time intervals in order to understand the attributes.	Learners can compare and order objects according to their measurable attributes. For example, a learner can directly compare the length of two sticks to say which is longer.
• use repetition of a single unit to measure length, area, volume and capacity, weight, angle, temperature and time.	Learners can use repeated standard units (such as centimetres) or non-standard units (such as hand spans) to measure the attributes of objects.
• select and use sensible units (both informal and standard or formal units) to measure length, area, volume and capacity, weight, angle, temperature, power and time • use common benchmarks to select appropriate methods for estimating measurements • carry out simple unit conversions within a measurement system.	Learners select and use appropriate standard units and instruments to measure length, weight, capacity and volume, angle, temperature, power and time. For example, a learner uses an electronic scale to weigh 200 grams of butter for a cooking recipe. The appropriateness of the unit depends on the problem or task. For example, to measure weight, appropriate units may include: – tonnes (for weighing a truck) – kilograms (for weighing a person) – grams (for weighing a bird). Learners use their knowledge of place value and the metric system to carry out simple conversions. Examples include: – 100 centimetres = 1 metre – 2,000 millilitres = 2 litres – 60 minutes= 1 hour – 3,000 watts = 3 kilowatts. Learners use common benchmarks to select appropriate methods for estimating measurements. For example, a hand span can be used as an estimate for 20 centimetres, or a pace can be used as an estimate for 1 metre.
• select and use sensible units and tools and/or formulas to measure the side lengths, perimeters and areas of rectangles, circles and triangles to appropriate levels of precision • carry out conversions within a measurement system.	Learners can calculate the area and perimeter of rectangles, triangles and circles from measurements of length. – A rectangle with side lengths of 7 centimetres and 6 centimetres has an area of 42 square centimetres. – A rectangular sports field that is 80 metres wide and 125 metres long has an area of 80 x 125 = 10,000 square metres. This is also called 1 hectare. Learners can convert units within measurement systems. – 1.25 litres is 1,250 millilitres. – 60 watts is 0.06 kilowatts. – 1 million seconds is about 11.5 days.
• select and use sensible units and tools and/or formulas to measure surface areas and volumes of prisms, including cylinders, to appropriate levels of precision • carry out conversions within and between measurement systems.	Learners can use appropriate units, tools and formulas to measure the surface areas and volumes of prisms, including cylinders. For example if a prism has side lengths of 4 centimetres, 6 centimetres and 7 centimetres, it has a volume of 4 x 6 x 7 = 168 cubic centimetres. Learners can convert between measurement systems. – 2.5 inches is 0.0635 metres or 6.35 centimetres. – a 60-watt light bulb running for 7 days uses 0.06 x 24 x 7 = 10.08 kilowatt hours of energy.

Further information

What is numeracy?

The term 'numeracy' is relatively new. Numeracy has been interpreted in different ways, mostly because of the very different needs of the users of the term.

The view of numeracy that is used for these learning progressions places an emphasis on the need for learners to gain:

- knowledge and understanding of mathematical concepts
- the ability to use their mathematical knowledge to meet the varied demands of their personal, study and work lives.

The numeracy learning progressions are based on the belief that, in order to meet the demands of being a worker, a learner and a family and community member, adults need to use mathematics to solve many different kinds of problems.

Key concepts

Several key concepts can be identified as central to the understandings about numeracy and about adult learners that have informed the development of the numeracy learning progressions. These concepts are covered below, under the following headings:

- Meaningful contexts and examples
- Understanding and reasoning
- Degree of precision
- Algorithms.

Meaningful contexts and examples

By using familiar, real-life contexts and examples, the numeracy learning progressions can raise learners' awareness of the mathematics all around them – and of the mathematical knowledge, skills and strategies they already possess.

Understanding and reasoning

The demands for adult numeracy arise from three main sources: community and family, the workplace and further learning. While each of these sources is likely to require different mathematical skills at varying levels, all mathematics needs to be learned with understanding so the learner can apply what they learn in a variety of situations.

Knowing some mathematical facts or routines is not enough to enable learners to use that knowledge flexibly in a wide range of situations. Being able to do mathematics does not necessarily mean being able to use mathematics well for solving real-life problems. A learner who counts decimal places to decide the number of decimal places in an answer without understanding the number operation involved may get 0.7 x 0.5 correct, but 0.7 + 0.5 incorrect. If the learner does not understand the mathematical processes involved, they will have no way of knowing why some of their answers are correct and others incorrect.

Degree of precision

When adults need to use mathematics to solve real-life problems, there is generally a certain amount of flexibility around the degree of precision necessary. In order to choose the best approach to solving a problem, an adult needs to begin by deciding how precise the solution needs to be.

For example, a practical problem may involve working out how much carpet is needed to cover the floor of a room. As a real problem for an adult, solving this problem may involve first asking and answering some practical questions, for example:

- "How accurate do I need to be?"
- "What tools (such as a calculator, a measuring tape or pen and paper) should I use?"

Depending on their specific purpose in this situation, the adult judges the degree of precision that would be reasonable. This could vary from very precise (for ordering and cutting the carpet) to a rough estimate (for thinking about whether or not to re-carpet). The adult decides which measurement units and tools to use, based on how accurate the measurement must be. For example:

- "Will I use hand spans, strides or a tape?"
- "Should I measure in metres, centimetres or millimetres?"

Algorithms

Traditional algorithms are methods for working out answers to number problems that have been developed over time. They involve a sequence of steps in a procedure that can be followed to solve a problem. Algorithms form part of the numeracy learning progressions, but the progressions make it clear that learners who use algorithms and calculators also need to be able to decide if the answers they obtain are reasonable. If learners cannot do this, they will need to develop either a better understanding of the algorithm or an alternative approach to calculating.

The steps used in algorithms are known as renaming, trading, borrowing, carrying over and other similar terms. The traditional algorithms work for all numbers but are often not the most efficient or useful method of computing. Most often, algorithms in mathematics are associated with the vertical working form traditionally used to solve operational problems.

For example:

$$\begin{array}{r} 345 \\ \underline{x4} \\ 1{,}380 \end{array}$$

Standard algorithms are accurate and efficient, but their meaning is often unclear to learners. Steps such as borrowing, carrying, moving the decimal point and shorthand notations can be confusing to learners. They can result in 'buggy' procedures that the user has no way of fixing when solutions appear to be unreasonable. When adult learners try to use procedures that have 'bugs', they often become frustrated, and negative attitudes towards mathematics may be reinforced.

To those who have learned an algorithm, the process of simply following that familiar algorithm may be faster and feel more comfortable than thinking about other ways to understand and solve a new problem. Learners need to know that they can continue to use this preferred method as long as they are always able to check that the answer they have obtained is reasonable and makes sense for the actual problem they are solving.

Connections between the strands

The numeracy progressions in this book are called Make Sense of Number to Solve Problems, Reason Statistically and Measure and Interpret Shape and Space.

The progressions in the Make Sense of Number to Solve Problems strand are related to progressions in the other strands in two ways. First, the other strands provide content or contexts that interact with and enhance the development of number. Second, the content of the other strands is directly affected by how well number concepts have been developed.

Additional details for each strand

Make Sense of Number to Solve Problems

In order to meet the demands of being a worker, a learner and a family and community member, adults need to be able to solve operational problems with numbers.

Three of the progressions in the Make Sense of Number to Solve Problems strand focus on number strategies, and three focus on key aspects of number knowledge. Number strategies are the processes that learners use to solve operational problems with numbers – strategies that make it easier to solve number problems with understanding. Rather than a single strategy for subtracting (or any operation), the most appropriate strategy can and should change flexibly as the numbers and the content or problem change. The three number knowledge progressions describe the key items of knowledge that people need to understand. They include number sequences, place value and number facts. The strategy and knowledge progressions are viewed as interdependent, with strategies creating new knowledge through use and knowledge providing the foundation for developing new strategies.[4]

Number strategies

> Number strategies can be grouped into counting strategies and partitioning strategies.

Counting strategies

Counting strategies involve counting in ones to solve problems, often with the support of objects (such as fingers).

Counting all the objects

This involves joining or separating sets to solve addition or subtraction problems. Learners count all the objects in both sets to find the answer.

Counting on

Learners count on or back to solve addition or subtraction problems. For example, instead of counting all objects to solve 8 + 5, a learner counts on from 8: *9, 10, 11, 12, 13.*

Partitioning strategies

Partitioning strategies are based on using knowledge of number properties to split numbers (partitioning) and combine them again in ways that make it easier to reach the solution. Partitioning strategies include the following strategies.

Deriving from known facts

Learners derive unknown information from a known fact. A learner may solve 25 + 26 by using what they know (25 + 25 = 50), then adding 1 to reach 51. Similarly, if a learner knows 6 x 7 = 42, they can solve 6 x 70 = 420.

Place-value partitioning

Learners break the numbers into ones, tens and hundreds, add numbers of the same place value together and then combine these numbers. For example:

63 + 35 can be solved as (60 + 30) + (3 + 5) = 98.

45 x 6 can be solved as (40 x 6) + (5 x 6) = 270.

Using tidy numbers with compensation

Learners round a number to the nearest ten or hundred, then compensate for what has been added or subtracted. For example,

73 – 29 can be solved as 73 – 30 + 1 = 44

and 64 + 28 can be solved as 64 + 30 – 2 = 92.

Using reversibility

Learners change a subtraction problem into an addition problem in order to have an easier route to the solution. For example,

66 – 48 becomes 48 + ? = 66; 48 + 2 + 16 = 66, so 66 – 48 = 18.

Halving and doubling (or dividing by 3 and trebling)

Learners use knowledge of number doubling and trebling. For example,

16 x 8 can be solved as 2 x (8 x 8) = 2 x 64 = 128.

16 x 4 can be solved as 8 x 8 = 64.

3 x 27 can be solved as 9 x 9 = 81.

Reason statistically

In order to be an informed citizen, employee and consumer, an adult needs to be able to reason statistically.

Preparing, analysing and interpreting data

The amount of statistical information available to help people make decisions in business, politics,

4 Ministry of Education, 2007.

research and everyday life is vast. For example, consumer surveys guide the development and marketing of products, experiments evaluate the safety and efficacy of new medical treatments and statistics sway public opinion on issues and represent (or misrepresent) the quality and effectiveness of commercial products.

Current thinking in statistics education emphasises the need for learners to undertake statistical investigations themselves in order to understand statistics and use them wisely. There are two main types of statistical investigation. In the first type, learners pose questions, gather data and use the data to answer the questions. In the second type of investigation, learners look for patterns and trends in existing data sets and generate questions to be answered. It is the second type of investigative approach that is addressed in these learning progressions. The decision to focus on existing data sets reflects the fact that most adults are seldom engaged in data collection but often need to consider data that has already been collected and presented.

Probability

Probability impacts on people's everyday decision-making in such varied contexts as buying a Lotto ticket, purchasing a car, taking medicine or taking an umbrella to work. (What are the chances of winning Lotto, surviving a crash in that particular model of car, experiencing one of the listed side effects of a certain drug or of the forecasted rain eventuating?) Human nature means that we don't always make decisions based on facts; intuition and wishful thinking often influence decisions that could be assisted by using a basic knowledge of probability. Terms often used for probability include *chance, luck, likelihood, odds, percentage* and *proportion*. Probability is often counterintuitive in the way it operates, so it is important that people do not assume that their initial assessment of a probability situation takes all the relevant factors into account and relates them correctly.

5 Steinback et al., 2003.

Measure and Interpret Shape and Space

> Measures are a cornerstone of mathematics and of our lives: it is difficult to think of anything that is not measured.
>
> Baxter et al., 2006, page 6

Measurement exists in all human cultures along with counting, locating, designing, explaining and playing.

Understanding what a measurable attribute is and becoming familiar with the units and processes that are used in measuring attributes is a major emphasis in the measurement progression. Experiences with measurement build understanding, making adults more aware of the dimensions of the world.[5] Measurement also offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas and statistical concepts.

The approach to measurement is a practical one, like the approach taken in the other numeracy progressions. The measurement progression emphasises the appropriateness and precision of the measure to the particular measurement problem or task.

An understanding of geometry and a sense of space are fundamental components of numeracy. Adults use ideas of shape and space when representing and solving problems in real-world situations and in other areas of mathematics. Geometric representations can help people make sense of area and fractions, while the shapes and patterns in histograms and scatter plots can give insights about data.

Adults use spatial reasoning when following maps, planning routes, designing floor plans and creating art. The Shapes and Transformations progression and the Location progression are more about describing relationships and reasoning than about definitions and theorems.

References

For a fuller reference list including the key works that underpin this document, refer to the reference list in the companion booklet, *Learning Progressions for Adult Literacy and Numeracy: Background Information.*

Baxter, M., Eamonn, L., Richards, L., Tomlin, A., Wresniwiro, T. and Coben, D. (2006). *Measurement Wasn't Taught When They Built the Pyramids – Was It? The Teaching and Learning of Common Measures in Adult Numeracy*. London: National Research and Development Centre for Adult Literacy and Numeracy.

Ministry of Education (2007). *Book 1: The Number Framework*. Wellington: Ministry of Education.

National Institute for Literacy (http://eff.cls.utk.edu). http://eff.cls.utk.edu/fundamentals/standard_use_math.htm

Steinback, M., Schmitt, M. J., Merson, M. and Leonelli, E. (2003). "Measurement in Adult Education: Starting with Students' Understandings". In D. Clements and G. Bright (eds), *Learning and Teaching Measurement: 2003 Yearbook* (pp. 318–331). Reston, Va.: National Council of Teachers of Mathematics.

Tertiary Education Commission (TEC) (2008a). *Learning Progressions for Adult Literacy*. Wellington: TEC.

Tertiary Education Commission (TEC) (2008b). *Learning Progressions for Adult Literacy and Numeracy: Background Information*. Wellington: TEC.

Glossary

Additive strategies	The methods used by learners to count, add, subtract, multiply and divide numbers. They include counting strategies and partitioning strategies.
Algorithm	A procedure that can be followed mechanically to find a solution.
Attribute	A quality or feature of something, for example, in a geometrical figure, "the opposite sides are parallel", or "all the angles are equal", or "it has line symmetry".
Axis (plural: axes)	A reference line. An axis of symmetry is any line along which a figure can be 'folded' so that one half fits exactly over the other.
Basic facts	The sums (additions) and products (multiplications) of all pairs of whole numbers from 0 to 9. These need to be memorised. The sums of single-digit numbers added to 10 can be called 'teen facts'. The sums obtained by adding single-digit numbers to whole numbers that are multiples of 10 can be called 'decade facts'. The facts for the sums of multiples of 10, 100, etc. (for example, 30 + 40, 500 + 600) are sometimes called 'place value facts'.
Bearings	The direction of one point as viewed from another, measured as an angle, clockwise from north; usually written as a three-digit number, for example, 015 degrees.
Bisector	A line that divides an angle or another line exactly in two.
Box plot (box-and-whisker)	A way of comparing two or more sets of data: a thin rectangular 'box' shows the extent of the interquartile range of the data (that is, from one-quarter to three-quarters), a line through the box shows the median; and the 'whiskers' extend beyond the rectangle to show the range of the data.
Category data/ categorical data	Data that categorise information according to some attribute (such as colour) rather than by measurement or counting.
Cognition	The process of acquiring knowledge. Cognitive skills are the skills used in acquiring knowledge.
Common factor	A whole number that divides exactly into two or more other numbers.
Compensating	Taking actions to offset the effects of other actions. For example, 18 + 25 can be solved using a strategy in which 2 is added to 18 to make the tidy number 20, and to compensate, 2 is subtracted from 25: 18 + 25 = 20 + 23 = 43.
Complementary events	One of a pair of events where one or other event must occur.
Congruence	Being identical in size and shape.
Counting all objects	A strategy for adding or subtracting groups of objects by counting each object, one at a time (compare this with counting on, a more sophisticated strategy that does not involve going back to 1).
Counting unit	Any numerical unit used as the basis for counting, for example, ones (1, 2, 3 …), tens (10, 20, 30 …).

Data	Information (obtained by measurement, counting or categorisation) that is used for reference or analysis.
Decade facts	Sums in which a single-digit number is added to a multiple of 10, for example, 30 + 3 = 33 (see also Basic facts).
Decimal	A number written with a decimal point, such as 6.25. The part before the point is a whole number amount, and the part after the point is a fraction less than 1 (this fraction has a denominator that is a power of 10, and its numerator is expressed by figures placed to the right of the decimal point, for example, 0.78, 0.5).
Denominator	The bottom part of a fraction, which indicates the number of equal parts a whole is divided into. For example, $^1/_6$, $^5/_6$, $^7/_6$, $^8/_6$ are all fractions of a whole divided into six equal parts.
Deriving from known facts	Using memorised number facts to find the answer.
Equal sharing	Working out how many times one number goes into another by repeated one-to-one matching. For example, to find out how many times 2 goes into 6, six items are divided into two equal piles.
Equivalent fractions	Fractions that represent the same value, such as $^8/_6$ and $^4/_3$.
Exponents	Powers, as in 10^3, which is a shorthand way of writing 10 x 10 x 10, or 1,000 (in this case, the exponent or power is 3).
Fraction	A numerical quantity that is not a whole number, for example, five-tenths, $^1/_4$, 0.45.
Histogram	A statistical graph in which data is represented by adjoining vertical bars. The bars are usually of equal width, in which case their height is a measure of frequency (that is, of 'how many' or 'how much').
Integers	All numbers in the set {… -3, -2, -1, 0, 1, 2, 3 …}. This set specifically excludes all fractional numbers.
Interquartile range	In statistics, the difference between the upper and lower quartiles. The quartiles can be found by lining up, in order of size, the set of all values to be included, then selecting the values that are one-quarter of the way from the bottom and one-quarter of the way from the top (see also Box plot).
Known fact	A number fact, such as 14 + 6 = 20, that the learner knows from memory (see also Deriving from known facts).
Line symmetry	A figure that has this attribute can be folded (at least metaphorically) onto itself, the two halves matching perfectly (line symmetry is also commonly known as reflective symmetry).
Mean	The average of a set of quantities.

Median	The middle number in a set of numbers that has been arranged in order of size.
Mental strategy	An in-the-head process the learner chooses to use to solve a problem.
Multiplicative strategies	The number strategies that learners use to solve multiplication and division problems. They include counting and partitioning strategies.
Multi-variate data	Data that contains information on three or more attributes or variables for each item in the set (for example, age, height and weight).
Mutually exclusive events	Events that cannot both occur.
Number knowledge	The key kinds of knowledge about number that learners need to know, including number identification, number sequence and order, grouping and place value and basic facts.
Number strategies	The counting and partitioning strategies learners use to estimate answers and solve number problems.
Numerator	The top part of a fraction; indicates the number of equal parts chosen. For example, the numerator in the fraction $^5/_2$ tells us this fraction is equivalent to five fractions of the $^1/_2$ kind.
Numerical data	Data that are expressed in number form (rather than by categories).
One-to-one counting	Counting in single numbers; counting by ones.
Partition	To split numbers into parts, for example, by place value (536 is five hundreds, three tens and six ones).
Partitioning strategies	Strategies that are based on splitting numbers into two or more parts and then combining them in a different way, for example, 26 + 9 can be partitioned into 26 + 4 + 5 and then combined as 30 + 5.
Percentage	A rate expressed as a number or amount out of each hundred.
Pictograph	A pictorial representation of some statistical information (often category data).
Place value	In our number system, the value of any digit depends on its place. For example, the place value of the 7 in the number 3,715 is 700, because the 7 is in the hundreds place.
Plane shape	A shape that has length and breadth but no depth; a two-dimensional shape.
Power of	The product obtained when a number is multiplied by itself a given number of times (the 'power' gives the number of times).
Prism	Any geometrical solid with identical parallel ends and straight sides.
Progression	A set of steps along a continuum, each step representing a significant learning development (refer to page 5).

Proportional reasoning — Reasoning that is based on comparing the relative size of objects (using multiplication or division) rather than their absolute size (using addition or subtraction).

Range — In statistics, the difference (found by subtraction) between the least and greatest values in a data set.

Reasonableness — A judgment about an answer based on the learner asking: "Bearing in mind the known details and the context, is the answer reasonable/plausible?"

Repeated addition — Adding the same number multiple times in order to find the answer to a multiplication problem. For example, finding the answer to 3 x 4 by saying 4 + 4 + 4 = 12.

Rotational symmetry — A figure with rotational symmetry fits exactly onto itself more than once as it is rotated through a complete turn.

Scatter plot — A graph on which the values of two variables are plotted as points. The pattern of the points suggests what kind of relationship (if any) exists between the two variables.

Skip-count — To count in regular amounts, skipping the intervening numbers, for example, counting in threes: 3, 6, 9 …

Spread — In statistics, the extent to which data are clustered round some central value.

Stem-and-leaf plot — A way of ordering data in order of size, from least to greatest value. Two-digit numbers are sorted first by the tens digit and then by the ones digit.

A data set consisting of the numbers 12, 67, 5, 20, 10, 17, 22 and 78 can be arranged in a table in which the tens digits are the stems and the ones digits the leaves:

Figure 1: stem-and-leaf plot example

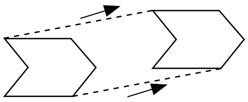
Stem	Leaf
0	5
1	0, 2, 7
2	0, 2
6	7
7	8

Strategies — Knowledge, skills and/or awareness combined and used for a particular purpose.

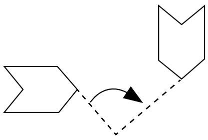
Teen facts — Sums in which a single-digit number is added to 10. For example, 10 + 3 = 13 (see also Basic facts).

Tidy numbers — Numbers that end in a 0 (10 and multiples of 10).

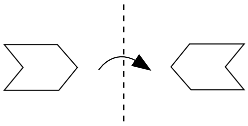
Time-series data	A set of observations, usually measurements or counts, ordered by time (for example, a shop's daily takings for each day in May).
Transformations	Changes in position and size of a shape. Translation (or slide) is the movement of a shape so that all its points move the same distance in the same direction.



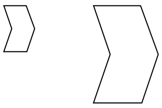
Rotation (or turn) is the movement of a shape when it is turned through an angle about a point in a plane.



Reflection (or flip) is the movement of a shape when it is reflected (or flipped) over a line in the plane of the figure.



Dilation is an enlargement or reduction of a shape in which all the linear measures are multiplied by the same number (called the scale factor).



Tree diagram	A diagram that systematically represents all outcomes for a sequence of events.
Unit fraction	Any fraction that has a numerator of 1, for example, $^1/_3$, $^1/_9$, $^1/_2$.
Variable	A symbolic quantity, often represented by a letter such as x, that may take many values, for example, personal income, height.
Vertex (plural: vertices)	Each angular point ('corner') of a polygon, polyhedron or other figure.
Whole numbers	All numbers in the set {0, 1, 2, 3 …}. This set excludes all negative and all fractional numbers.

Strand charts

Make Sense of Number to Solve Problems

	ADDITIVE STRATEGIES PROGRESSION	MULTIPLICATIVE STRATEGIES PROGRESSION	PROPORTIONAL REASONING STRATEGIES PROGRESSION
	MOST ADULTS WILL BE ABLE TO:	MOST ADULTS WILL BE ABLE TO:	MOST ADULTS WILL BE ABLE TO:
	• solve addition and subtraction problems by counting all the objects.	• solve multiplication problems by counting all the objects.	
	• solve addition and subtraction problems by counting on or counting back, using ones and tens.	• solve multiplication problems by skip-counting, often in conjunction with one-to-one counting and often keeping track of the repeated counts by using materials (for example, fingers) or mental images.	• find a fraction of a set by using equal sharing.
	• solve two-digit by one-digit addition and subtraction problems mentally, using partitioning strategies.	• solve single-digit multiplication and division problems mentally, using known multiplication facts and repeated addition.	
	• solve multi-digit addition and subtraction problems, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	• solve multiplication and division problems with single-digit multipliers or divisors mentally, using partitioning strategies and deriving from known multiplication facts.	• use known multiplication and division facts to find fractions of a whole number.
	• solve addition and subtraction problems involving decimals and integers, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	• solve multiplication or division problems with multi-digit whole numbers, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	• use multiplication and division strategies to solve problems that involve simple equivalent fractions and simple conversions between fractions, decimals and percentages.
	• solve addition and subtraction problems involving fractions, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	• solve multiplication or division problems with decimals, fractions and percentages, using partitioning strategies *or alternatively* • justify the reasonableness of answers to problems solved, using a calculator or algorithm.	• use multiplication and division strategies to solve problems that involve proportions, ratios and rates.

NUMBER SEQUENCE PROGRESSION	PLACE VALUE PROGRESSION	NUMBER FACTS PROGRESSION
MOST ADULTS WILL KNOW:	MOST ADULTS WILL KNOW:	MOST ADULTS WILL KNOW:
• the sequence of numbers, forwards and backwards, to at least 20.		• addition facts with sums of 5 or 10 and the decade facts.
• the sequence of numbers, forwards and backwards, to at least 100 • how to skip-count in twos, fives and tens to 100.	• 10 as a counting unit, the tens in numbers to 100 and the place values of digits in whole numbers up to 100.	• basic addition and subtraction facts up to 10 + 10.
• the sequence of numbers, forwards and backwards, to at least 1,000 • the number that is 1, 10 and 100 before or after a given number in the range 0–1,000 • how to skip-count in twos, threes, fives and tens to 1,000 • how to order fractions with like denominators.	• the tens and hundreds in numbers to 1,000 and the place values of digits in whole numbers up to 1,000.	• basic multiplication and division facts up to 10 x 10.
• the sequence of numbers, forwards and backwards, by ones, tens, hundreds and thousands, to a million • how to give the number 1, 10, 100 or 1,000 before or after a given number in the range 0–1,000,000 • the sequence of decimal numbers in tenths and hundredths • how to order unit fractions.	• how many tens, hundreds and thousands there are in any whole number • that 10 tenths make one whole.	• basic multiplication facts with tens, hundreds and thousands • fraction and decimal groupings that make 1.
• the sequences of integers, fractions, decimals and percentages, forwards and backwards, from any given number.	• how many tenths, hundredths and thousandths are in any number, including decimal numbers • how to convert percentages to decimals and vice versa • what happens when a whole number or decimal is multiplied or divided by a power of 10.	• common factors of numbers up to 100 • fraction, decimal and percentage conversions for halves, thirds, quarters, fifths and tenths • the convention for exponents.

Reason Statistically

PREPARING DATA FOR ANALYSIS PROGRESSION	ANALYSING DATA FOR INTERPRETATION PROGRESSION
MOST ADULTS WILL BE ABLE TO:	MOST ADULTS WILL BE ABLE TO:
• sort objects according to their attributes, organise data about the objects and represent data, using concrete objects or pictures.	• describe parts of the data and the set of data as a whole to determine what the data show.
• sort and organise category data and represent it, using tables, pictographs and bar graphs.	• describe the general features of a data set.
• sort, organise and represent data, using tables and graphs such as line plots, bar graphs and line graphs • recognise the differences involved in representing category and numeric data.	• describe the shape and important features of a sample data set (considering especially median and range) • compare two or more samples.
• sort, organise, clean and represent multi-variate data, making appropriate use of histograms, stem-and-leaf plots, box plots (box-and-whisker diagrams) and scatter plots • graph time-series data.	• find, use and interpret measures of centre and spread, including mean and interquartile range.

INTERPRETING DATA TO PREDICT AND CONCLUDE PROGRESSION	PROBABILITY PROGRESSION
MOST ADULTS WILL BE ABLE TO:	MOST ADULTS WILL BE ABLE TO:
	• identify all possible outcomes in situations that involve simple (single-stage) chance • use words to describe the likelihood of particular outcomes (events).
• make sensible statements based on the general features of a data set.	• use fractions to express the probability of events • recognise uncertainty in simple (single-stage) chance situations.
• draw conclusions and make predictions, based on evidence from the data.	• use relative frequency to provide an estimate of the probability of an event • use fractions, ratios and percentages to express probabilities • compare the results of trials or observations with expectations based on models.
	• determine the probabilities in simple multi-stage probability situations • apply the law of large numbers to probability situations.
• use observations based on samples to make conjectures about the populations from which the samples were taken.	• determine the probabilities in more complex multi-stage chance situations • apply the notion of 'expected value' to probability situations.

Measure and Interpret Shape and Space

	SHAPES AND TRANSFORMATIONS PROGRESSION	LOCATION PROGRESSION
	MOST ADULTS WILL BE ABLE TO:	MOST ADULTS WILL BE ABLE TO:
	• sort and describe objects by their shape attributes.	• describe, name and interpret relative positions in space.
	• identify and describe plane shapes in objects • record the results of transformations (flips, turns and slides) on plane shapes.	• give and follow instructions for movement that involve distance and directions.
	• create mental images of plane shapes • recognise and represent plane shapes in objects from different perspectives • predict and communicate the results of transformations (flips, turns, slides and/or scaling) on plane shapes.	
	• define plane shapes and prisms by their spatial features • create and describe mental images of prisms, including cylinders • make two-dimensional representations of prisms (and vice versa) • describe the transformations (flips, turns, slides and/or scaling) that are used to map one object onto another.	• use grid co-ordinate systems to specify locations and to describe routes.
	• define classes of plane shapes by their geometric properties and classes of solid shapes by their surfaces • use spatial visualisation to solve problems that involve surface area and volumes of prisms • describe sizes, positions and orientations of shapes under transformation (flips, turns, slides and/or scaling).	• communicate and interpret locations, directions and distances, using bearings, grid references and scales.
	• visualise three-dimensional objects and spaces from different perspectives and analyse their cross-sections • examine the congruence, similarity and line or rotational symmetry of objects, using transformations.	

MEASUREMENT PROGRESSION
MOST ADULTS WILL BE ABLE TO:
• compare and order objects directly, using attributes of length, area, volume and capacity, weight, angle, temperature and time intervals in order to understand the attributes.
• use repetition of a single unit to measure length, area, volume and capacity, weight, angle, temperature and time.
• select and use sensible units (both informal and standard or formal units) to measure length, area, volume and capacity, weight, angle, temperature, power and time • use common benchmarks to select appropriate methods for estimating measurements • carry out simple unit conversions within a measurement system.
• select and use sensible units and tools and/or formulas to measure the side lengths, perimeters and areas of rectangles, circles and triangles to appropriate levels of precision • carry out conversions within a measurement system.
• select and use sensible units and tools and/or formulas to measure surface areas and volumes of prisms, including cylinders, to appropriate levels of precision • carry out conversions within and between measurement systems.

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