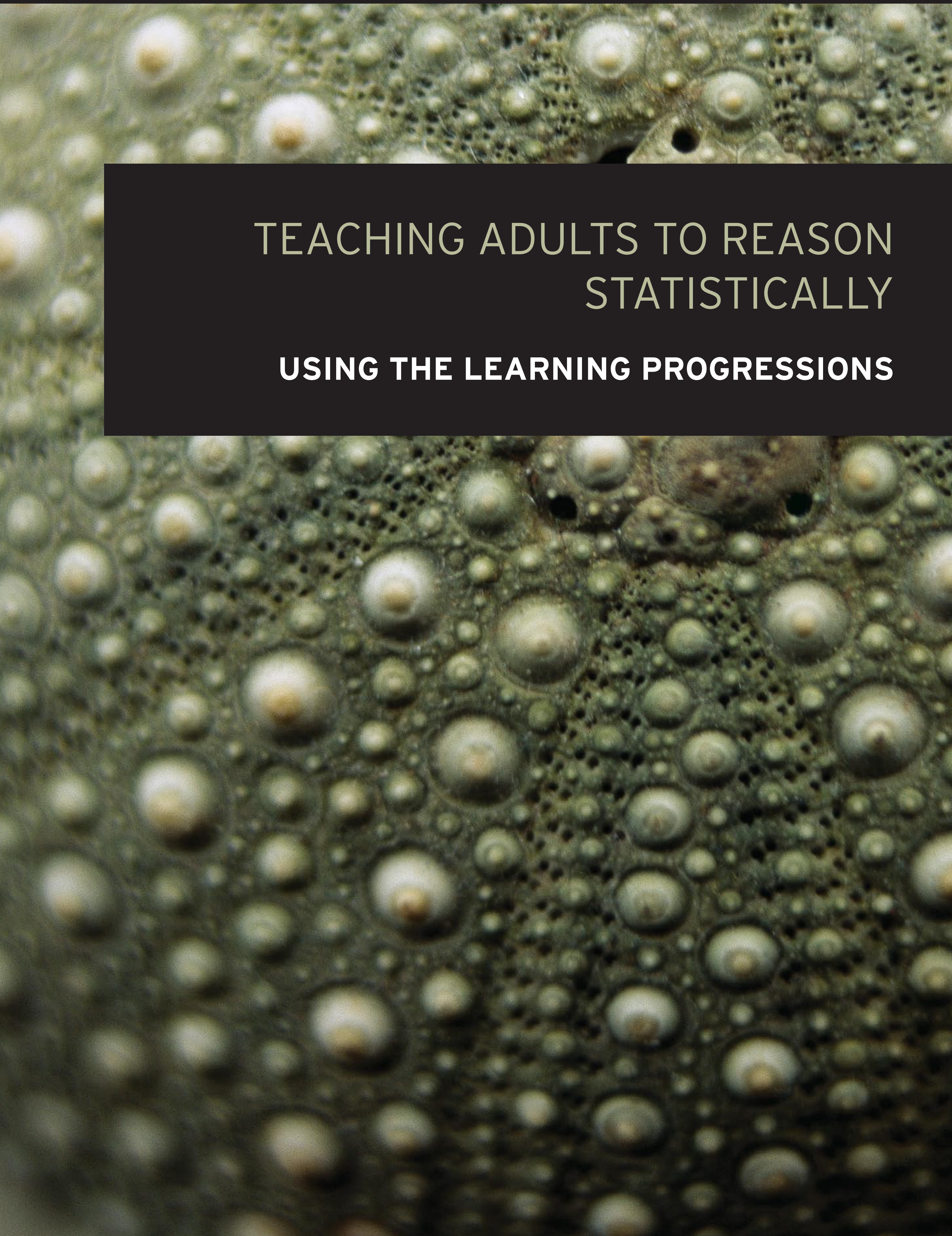
Tertiary Education Commission
Te Amorangi Mātauranga Matua

TEACHING ADULTS TO REASON STATISTICALLY

USING THE LEARNING PROGRESSIONS



Mā te mōhio ka ora:
mā te ora ka mōhio

Through learning there is life:
through life there is learning!

The Tertiary Education Commission would like to thank the many people who have contributed their time and expertise to this document, in preparation, design and development, consultation and review.

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Introduction

Teaching Adults to Reason Statistically: Using the Learning Progressions is part of a set of resources developed to support the teaching of literacy, language and numeracy for adult learners. The end goal is to enable tutors to meet the learning needs of their adult learners so those learners can engage effectively with the texts, tasks and practices that they encounter in their training and learning. The suggestions in each booklet are aligned with the following Tertiary Education Commission (TEC) publications:

- *Learning Progressions for Adult Literacy and Numeracy: Background Information*
- *Learning Progressions for Adult Literacy*
- *Learning Progressions for Adult Numeracy.*

These can be located on the TEC website at www.tec.govt.nz.

These resources are based on research into effective adult literacy and numeracy, as described in *Lighting the Way*.[1] They also draw on school-sector work in literacy and numeracy, including Numeracy Project publications and the teachers' books *Effective Literacy Practice in Years 5 to 8* and *Effective Literacy Strategies in Years 9 to 13*.[2]

Readers are referred to the learning progressions publications (as listed above) for more detailed discussions of adult learners, ESOL learners and the theoretical basis for each of the progressions. These books also contain glossaries and reference lists.

This set of resources has been developed to support the learning progressions. The suggestions are initial ideas only: they are aimed at helping tutors apply the learning progressions to existing course and learning materials. It is expected that tutors will use, adapt and extend these ideas to meet the needs of learners and their own teaching situations. There are many other resources available for tutors to use, and comparisons with the learning progressions will help you determine where other resources may fit in your programmes, and how well they might contribute to learner progress.

1 Ministry of Education (2005). *Lighting the Way*. Wellington: Ministry of Education.
2 Ministry of Education (2006). *Effective Literacy Practice in Years 5 to 8*. Wellington: Learning Media Limited.
Ministry of Education (2004). *Effective Literacy Strategies in Years 9 to 13*. Wellington: Learning Media Limited.

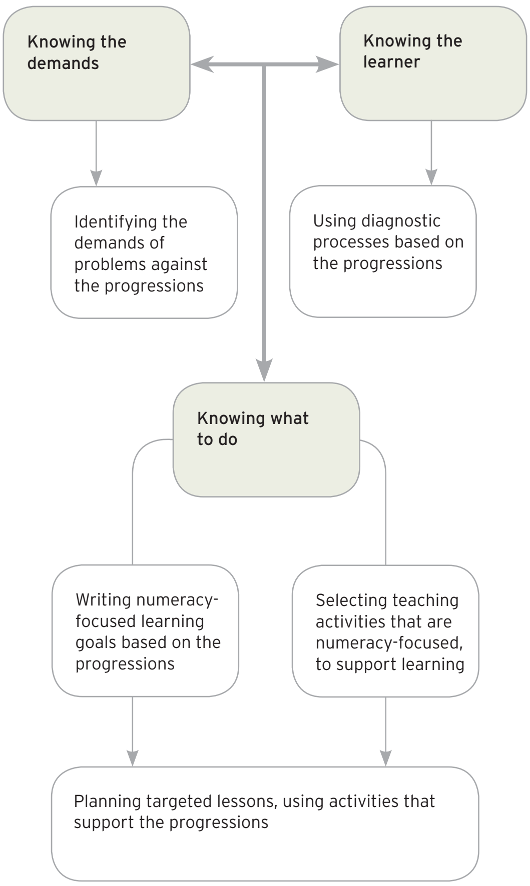
How to use this resource

There are three main sections in this resource:

- Knowing the demands (of the problems that learners want or need to manage).
- Knowing the learner (what they can do already, in order to determine the next learning steps).
- Knowing what to do (to help learners move on to the next steps).

These sections fit a process that can be illustrated as a flow chart.

Teaching adults to reason statistically: using the learning progressions



It is not essential to follow this order – in some circumstances, it will make sense to start by getting to know the learners before finding out what it is that they want to be able to do.

The following guide to working with this resource should be used alongside the information in *Learning Progressions for Adult Numeracy*.

Knowing the demands

First, identify the numeracy demands of the course, task, problems or work practices the learners want to accomplish and map these against the learning progressions.

In this section, you will find an example of a problem that has been analysed (mapped) against the *Reason Statistically* progressions. This mapping shows the progressions and steps the learners would need to be working at in order to solve the problem.

Knowing the learner

Use the tools in this section and the learning progressions to identify the learners' skills. This section contains diagnostic activities and questions that can be used to identify the learners' strengths and needs in this area of numeracy.

Knowing what to do

Use the learning progressions to set achievable goals for and with the learners. The decisions about what to teach should be based on the identified demands of the tasks and where learners 'sit' on the learning progressions. Identify specific activities and materials to use (based on your course and context), then apply them in your teaching. Finally, review and reflect on the outcomes for the learners, with the learners.

In this resource, mapping the problems the learners will encounter is the first step in planning for instruction. The next step is finding out where the learners 'sit' on the progressions. Where there is a gap between what the learners can do and what a task demands, you and your learners can refer to the learning progressions to make decisions about what to teach and learn next.

Strands and progressions

The learning progressions are organised within seven strands that cover the key components of listening, speaking, reading, writing and numeracy. Each progression shows a series of steps that reflects the typical sequence of skill development for oral language, written language and numeracy. The steps described are not tasks to be mastered in a set order. They do, however, offer information and a structure that can be used to develop curricula and learning and assessment tools. This current resource provides examples of how the progressions can be used. You are encouraged to design your own materials for teaching and learning to meet the needs of the adults with whom you work.

It is important to keep in mind that although the progressions are described in separate strands, in practice we use literacy, language and numeracy skills and knowledge in ways that are typically interconnected. For example, a person may **listen** to a report about rising interest rates, **speak** to their partner about their mortgage, **read** the information from several banks (using their knowledge of **numbers** to interpret and compare rates), then **write** questions to ask a bank about the options for managing a mortgage. Even filling in a form requires both reading and writing skills, and may also involve a discussion to clarify terms or requirements. Learners will better understand how their existing knowledge can support new learning when these connections are made clear.

Knowing the demands

Adult learners need to learn to interpret and solve particular kinds of statistical problems for their individual purposes. As their tutor, you have to be able to analyse the problems that your learners need to solve and identify the demands and supports that such problems present to the learners. This section provides a guide to mapping the kinds of problems that adult learners need to be able to solve in relation to the learning progressions.

Mapping problems against the progressions

To determine the challenges of problems the learners are expected to solve, you need to compare typical examples of these problems with the *Reason Statistically* progressions and make decisions about where each problem fits with the relevant progressions. By comparing this information with what you know about the learners' knowledge, skills and strategies, you will be able to determine the priorities for teaching and learning.

The example offered in this section is a model to help you work out how to analyse the problems your learners will need to solve. Not every problem needs to be analysed in such detail, but it would be worthwhile analysing problems that are fundamental to a course.

Mapping process

The general process for all numeracy strands is to:

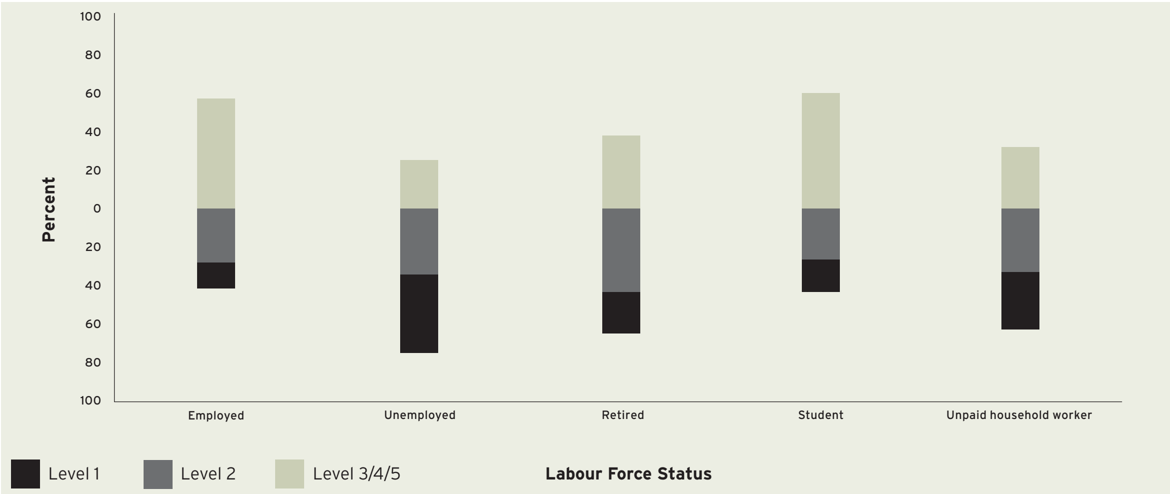
- identify the strand or strands involved
- identify the progression or progressions involved
- identify the appropriate step in each applicable progression

Example of a mapped problem

Numeracy and labour force status

The figure on page 7 shows the numeracy levels for different groups in New Zealand. These are anchored at the boundary of levels 2 and 3 to allow comparison of 'low numeracy' (levels 1 and 2) or 'higher numeracy' (levels 3, 4 and 5).

Numeracy levels on the National Qualifications Framework



Data sourced from: http://www.educationcounts.govt.nz/__data/assets/pdf_file/0019/16750/Factsheet_2-final.pdf

PROBLEM	SOLUTION	PROGRESSION(S) AND STEP(S)
What does this figure tell you about the numeracy levels of the labour force?	About 60 percent of students have 'higher numeracy' levels.	*Analysing Data for Interpretation*, 3rd step
	The employed and student populations are skewed toward the higher end of the numeracy spectrum, while the populations of the unemployed and unpaid household workers are skewed toward the lower end.	*Analysing Data for Interpretation*, 4th–5th steps

Knowing the learner

General guidelines for using the diagnostic questions

- The aim of the statistics diagnostic activity and questions is to give a general picture of a learner's ability to understand and critically reflect on statistical information they may be presented with in their daily lives.
- Two of the progressions (*Analysing Data for Interpretation* and *Interpreting Data to Predict and Conclude*) are assessed by using a diagnostic activity (glasses of water) and diagnostic questions.
- The *Probability* progression is assessed by a series of diagnostic items and questions.
- There are strong links between all of the progressions, and information from the diagnostic activity and questions will help you determine the next teaching and learning steps for the learners on all of the progressions.
- If the learner is finding the questions too difficult, it is not necessary to ask all of the questions.
- The diagnostic activity and the questions relate to authentic data. Ensure the learners understand all the language used.

At this stage, there has been no development of diagnostic assessment items for the *Preparing Data for Analysis* progression. The reasons for this include the following:

- The skills and concepts contained in this progression are less frequently taught in adult learning classes than the other progressions.
- The concepts or skills associated with this progression are difficult to assess in a short time period. It is also believed that the other two data-related progressions give you sufficient information to assess the learning needs of the learners.

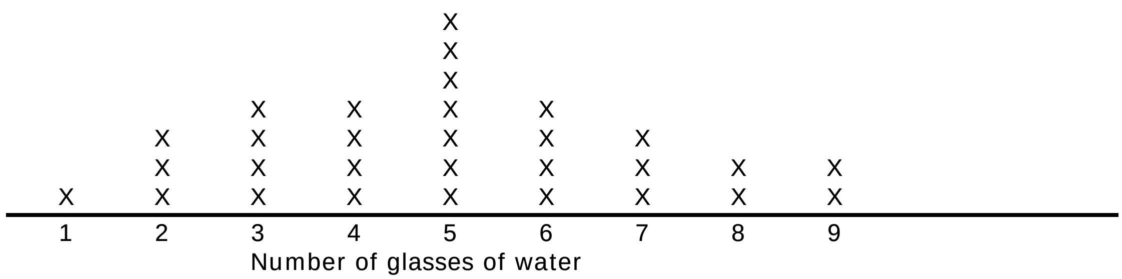
Diagnostic activity

The learners are asked a series of questions which probe their understanding of the information presented in a dot plot and bar graph. The diagnostic questions increase in complexity as you move to the more advanced steps of the progression.

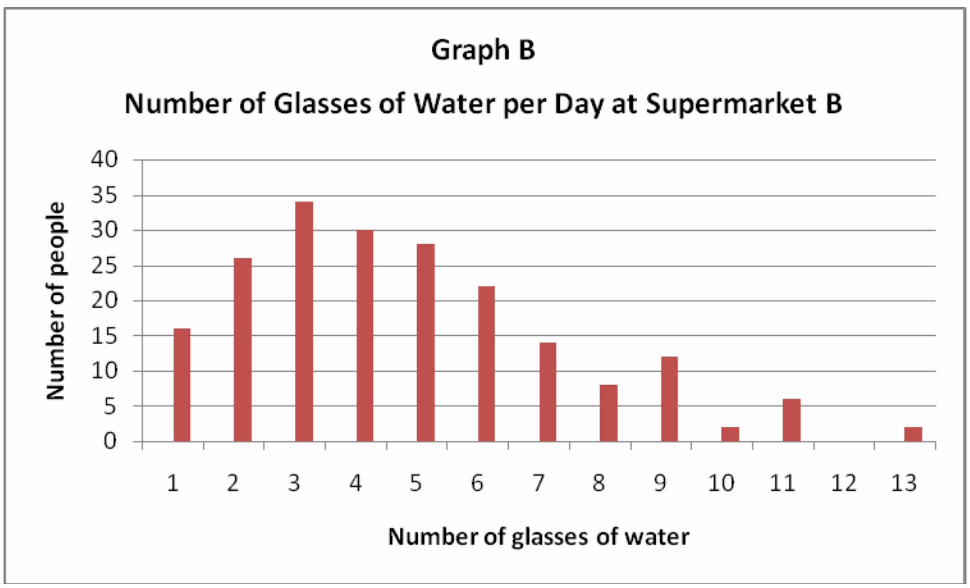
Resources needed

- A4 copy of Number of Glasses of Water a Day dot plot (Graph A)

Graph A
Number of Glasses of Water per Day at Supermarket A



- A4 copy of Number of Glasses of Water a Day bar graph



Instructions

Begin by explaining to the learners that at Supermarket A 30 people were asked how many glasses of water they drank in a day. The information collected is shown in Graph A (the dot plot). At Supermarket B 200 people were asked the same question. The information collected is shown in Graph B (the bar graph). The survey was trying to establish whether New Zealanders drink the recommended 8 glasses of water per day. Show the learners the graphs and tell them that this is what survey found. Give them time to look at the graphs before you pose the diagnostic questions.

Step	Diagnostic questions for *Analysing data progression*	Comments to the tutor
1 and 2	Show the learner Graph A. 1. How many people drank 6 glasses of water a day? 2. How many people drank 7 or more glasses of water a day? 3. How many glasses of water did the person who drank the most drink?	Q1 and Q3 check whether the learner can read information directly from a dot plot graph. Q2 checks whether the learner can read “between” the data. Q1 and Q2 check whether the learner can “lift” information from the body of the graph (frequency information). Q3 checks whether the learner can “lift” information from the axis.
3	Show the learner Graph B. 4. About how many people drink 5 glasses of water a day? 5. What does this graph tell you about how many glasses of water most people drink per day?	Q4 gives an idea of whether the learner can read information directly from a bar graph. Q5 checks if the learner has a sense that the data is clustered between 1 and 6 glasses. If the learner answers “3” then they do not understand that 34 people does not tell you about “most people”.
4-5	6. Approximately what do you think the median number of glasses is? 7. What differences do you notice between the two graphs?	Q6 checks if the learner understands that the median is the point which 50 % of the data lies above and 50% lies below. In this example the median is 3 glasses. Q7 checks if the learner can compare the centre and spread for two sets of data. A leaner that has this understanding will make comments like: The typical person in Supermarket A drank more than a typical person in Supermarket B (centre). At Supermarket B there was a wider range/more variation in the number of glasses drunk (spread). If a learner responds with an observation of very particular data, (for example At Supermarket A 1 person drank 1 glass of water but at Supermarket B 12 people did) encourage them to make comments of a more general nature.
6	8. Do you think it is better to use the median or mean to describe the centre of this data? Why?	Q8 checks if the learner understands that the median will describe the centre of the data better in this case. This is because, unlike the mean, the median is unaffected by outliers which are the small numbers of people who drink 9 or more glasses a day. If the learner says the median is better because the mean is unlikely to be a whole number, acknowledge this is correct but irrelevant.

Step	Diagnostic questions for *Interpreting data to predict and conclude* progression Note this progression starts at Step 3 All these questions relate to Graph B from Supermarket B	Comments to the tutor
3	1. Do you think that the people surveyed drink the recommended 8 glasses of water day?	Q1 checks whether the learners can conclude that the people surveyed do not on the whole drink 8 glasses of water a day.
4-5	2. The survey wanted to find out if New Zealanders drink the recommended 8 glasses of water day. Can you conclude this from the data? Why or why not?	Q2 checks if the learner understands that a sample of 200 people at a supermarket is not representative of the New Zealanders and therefore conclusions about New Zealanders as a whole cannot be made.
6	3. If you surveyed another 20 people and plotted their data on a new graph would it look the same as the first graph?	Q3 checks if the learner understands that the graph would not necessarily look the same because of the uncertainty and variability associated with small samples.

Diagnostic questions (Probability)

Resources needed: cards (see page 13)

DIAGNOSTIC ASSESSMENT ITEM	COMMENTS TO THE TUTOR
1. I have a pencil case with red, blue and green pens in it. What are all the possible outcomes if I reach in and select one pen? Answer: red, blue, green	Q1 checks whether the learners can give all possible outcomes and identify outcomes for an event.
2. If I reach, without looking, into a pencil case that contains 10 blue pens, and 1 red pen, which statement best describes my chance of selecting a blue pen. a. **Certain** to get a blue pen. b. **Very likely** to get a blue pen. c. **Probably** get a blue pen. d. **Impossible** to get a blue pen. Answer: b	Q2 checks whether the learners can express the likelihood of events, using words.
3. What is the chance of getting a 6 if you roll a standard six-sided dice? Answer: $^{1}/_{6}$	Q3 checks whether the learners can use fractions to express the probability of an event.
4. If I roll the dice and get a 3, do you think I have the same chance, a better chance or a worse chance of getting a 3 if I roll the dice again? Answer: same chance	Q4 checks whether the learners know that the outcome of the next event is not influenced by the previous event (previous roll of the dice).
5. Show Card A. Here is a table that shows results of a transport survey that asked 100 workers how they travelled to work. How likely do you think it is that the next worker surveyed travelled by car or bus? (a very good chance; a high chance; 85%, etc) Explain your reasoning. Answer: a high chance Ask the learners to express the likelihood that the worker walked to work as a fraction and a percentage. Answer: $^{5}/_{100}$, 5%	Q5 checks whether the learners can use frequencies to predict probabilities. This question also checks whether the learners can express probabilities with fractions, percentages and words.
6. Show Card B. Which bag gives the better chance of selecting a black ball? Explain your reasoning. Answer: same chance	Q6 checks whether the learners can use fractions to express probabilities and also whether the learners can use the relative size of a particular outcome rather than absolute size when comparing the likelihood of events. This means that the learners recognise that $^{3}/_{4}$ is the same as $^{6}/_{8}$ (or 1:3 is the same as 2:6) rather than focusing on the fact that there are 6 black balls in bag B compared to 3 in bag A.
7. What is the chance of drawing a King from a pack of 52 playing cards? Answer: $^{4}/_{52}$ or $^{1}/_{13}$ Does this mean that if I draw 13 cards I will definitely get a King? Why or why not? Answer: no	Q7 checks whether the learners know how to use fractions to identify a theoretical probability. The question also checks that the learners understand that while the theoretical probability is a way of predicting what *may* occur, it does not tell you what *will* occur as it is a chance event.

<table>
<tr><th>DIAGNOSTIC ASSESSMENT ITEM</th><th>COMMENTS TO THE TUTOR</th></tr>
<tr><td>
8. Dan is tossing a coin and a dice at the same time. What is the probability that he tosses a head and a 6?
Answer: $^1/_{12}$

Sample space, using an organised list:
H1, H2, H3, H4, H5, **H6**
T1, T2, T3, T4, T5, T6

Sample space using a two-way table
<table>
<tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr>
<tr><td>Heads</td><td></td><td></td><td></td><td></td><td></td><td>✓</td></tr>
<tr><td>Tails</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
</td><td>
Q8 checks whether the learners are able to calculate the theoretical probability of a simple two-stage event. Notice whether the learners use an organised list, a two-way table or multiplication ($^1/_6$ x $^1/_2$ = $^1/_{12}$) to solve the problem.
</td></tr>
<tr><td>
9. Sam reached into a bag of balls 10 times to pull out 1 ball each time, and each time he returned the ball to the bag. He took out 3 red balls and 7 blue ones. He said that there was a 30% chance of getting a blue ball.

Christy then pulled out 1 ball from the bag 100 times and returned the ball to the bag after each time. She got 53 blue balls and 47 red ones. She said there was about an even chance of getting a blue ball.

Who do you think is more likely to be correct? Explain your reasoning.
Answer: Christy
</td><td>
Q9 looks for evidence that the learners understand that 10 trials is not very good evidence of the probability and that 100 trials will tell more about the likelihood of an event happening.
</td></tr>
<tr><td>
10. There are two identical boxes. One box contains a real $10 note and 2 fake notes. The other box contains a real $10 note and 1 fake note. If you choose 1 box and then take 1 note from that box without looking, what is your chance of getting a real $10? Explain your reasoning.
Answer: $^5/_{12}$ (tree diagram)
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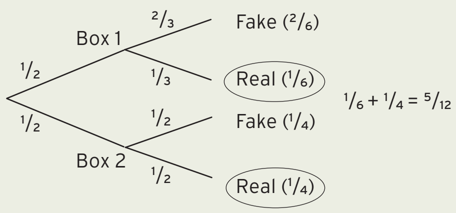
Q10 looks for evidence that the learners know that the events are dependent and that they have a method for correctly examining the event. (Note: If the learner gives an incorrect answer, check that the error is not just in calculating with fractions.)


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<tr><td>
11. Show Card C. How much should the insurance company charge as its average premium to break even in its costs for drivers aged 15 to 21?
Answer: $760
0.8 (0) + 0.1 (2,000) + 0.05 (4,000) + 0.03 (6,000) + 0.01 (8,000) + 0.01 (10,000) = 760
</td><td>
Q11 looks for evidence that learners are able to apply expected values to understanding probability situations such as insurance premiums.
</td></tr>
</table>

Resources for diagnostic questions

Card A

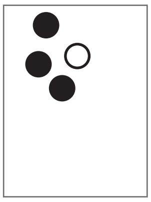
TRANSPORT SURVEY OF 100 WORKERS	
TRANSPORT METHOD USED	PERCENTAGE OF WORKERS
Walked	5
Car (passenger)	15
Car (driver)	30
Bus	40
Cycled	10

Card C

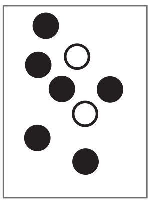
INSURANCE CLAIMS FOR DRIVERS AGED 15-21	
AMOUNT OF CLAIM (TO NEAREST $2,000)	PROBABILITY
0	0.80
$2,000	0.10
$4,000	0.05
$6,000	0.03
$8,000	0.01
$10,000	0.01

Card B

Bag A: 3 black and 1 white ball



Bag B: 6 black and 2 white balls



Which bag gives the better chance of picking a black ball?

a. Same chance

b. Bag A

c. Bag B

d. Don't know

Knowing what to do

General principles for guided teaching and learning activities

Each activity is aligned to a step on one of the *Reason Statistically* progressions and is intended to strengthen the learners' understandings of the concepts and skills associated with that step.

The activities assume the learners understand the concepts and skills from earlier steps in the progression, so it is important that you check the learners have these understandings.

Each teaching and learning activity includes the following components:

- A statement summarising the mathematical understandings that you will support the learners to develop in the activity.
- A list describing the teaching points covered in the activity.
- A list of resources or materials used in the activity. (These resources are readily available within adult learning settings and do not require the purchase of specialised materials.)
- A guided teaching and learning sequence that details the steps for you to use to develop the learners' knowledge of the skills and concepts addressed. The sequence includes questions for you to pose to the learners. The questions are intended to help the learners clarify their understandings as they explain their thinking to others. The questions are also designed to help you scaffold the learners' understanding of the concept or skill being explored.
- A follow-up activity for the learners to complete independently.

Summary of guided teaching and learning activities

	PREPARE DATA	ANALYSE DATA	INTERPRET DATA	PROBABILITY
				Describing likelihood page 31
	Sort, represent and interpret category data page 15			Understanding chance page 33
	Sort, represent and interpret number data page 18			Using frequencies to predict page 35 Theoretical probability page 38 Lotto winners page 40
		Understanding the mean I page 21 Understanding the mean II page 23 Sample sizes page 26	Understanding media reports that include statistical information page 29	Fair games? page 42

Sort, represent and interpret category data

Preparing Data for Analysis progression, 3rd step; Analysing Data for Interpretation progression, 3rd step; Interpreting Data to Predict and Conclude progression, 3rd step

The purpose of the activity

In this activity, the learners sort and organise category data and represent it on graphs in order to develop the ability to read and derive meaning from graphs created by themselves or others.

The teaching points

- Developing the ability to read and derive meaning from bar graphs is a developmental process, and this activity is designed to assist that development by moving from:
  - the learner's own data
  - the learner's data being represented by a sticky note placed on a graph
  - the learner's data being included in a bar on a graph.
- Working with category data (for example, favourite sport or TV programmes watched) is easier for the learners than number data (for example, number in your family, height, age, etc) and is therefore used in this initial activity.
- The learners need to choose categories in which to sort and organise the data.
- The scale chosen for a graph depends on the data.
- The purpose of statistics is to predict and tell a 'story', and when the learners are developing their understanding of statistics, it should always be in the context of a 'story', and that 'story' needs to be considered when preparing, analysing and interpreting data.
- Discuss with the learners the connections between statistics and everyday situations.

Resources

- Sticky notes.

The guided teaching and learning sequence

Choose a 'story to tell' that is of interest and relevant to the learners and generates category data. To have enough data, choose a 'story' where there is likely to be a wide range of responses or where you can ask each learner to give two or three responses. Examples could include:

"What do you do in your leisure time?"
"What type of television programme do you enjoy watching?"
"What do you read most?" (This question could serve a literacy purpose by encouraging the learners to recognise that reading is used in a greater variety of contexts than books and newspapers.)

1. Write the question on the board and ask the learners to discuss what they think the response to the question might be. Record their predictions on the board.
2. Ask the learners to answer the question by writing their responses on sticky notes (to be used later) and also by recording their response(s) on the board.
3. Ask the learners to consider whether recording and reporting every learner's response is the best way to answer the question. Listen for, and prompt if necessary, the idea that the responses could be summarised by sorting them into broader and fewer categories. (For example, for the question "What do you do in your leisure time?", responses might include: karate, soccer, yoga, netball, rugby, read, go to parties, chat on the net, and broader categories might include: team sports, individual sports, internet-based activities, etc).

continued...

4. Ask the learners to choose possible categories and share their choices. Discuss until there is agreement on categories. Make sure the number of agreed categories is less than the number of learners.

5. Draw a horizontal line on the board with the categories listed in a row underneath. Ask the learners to place their sticky notes above the appropriate category in such a way that they can easily compare the number of notes applied to each category. You may need to prompt the learners to place the sticky notes in a vertical line. Ask:

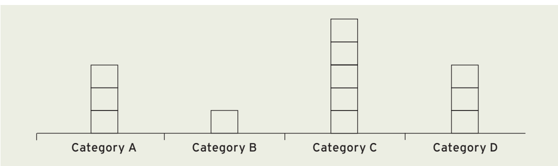
   "Can you compare the number in each category without counting them?"

   Listen for and reinforce the response that the height of the line indicates the number in each category. Ask:

   "Could your sticky notes be better placed to make the comparison more accurate?"

   Listen for and reinforce the response that the sticky notes and the spaces between them need to be the same size for accurate comparison.

   Ask the learners to adjust the positioning of the sticky notes if necessary.



6. Explain you are going to draw a bar graph that uses bars instead of the sticky notes to represent the responses.

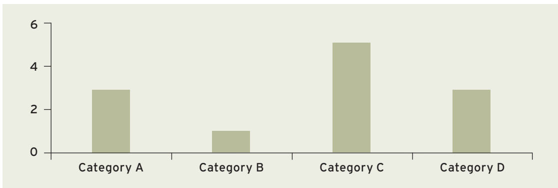
   Draw bars over the sticky notes. Ask:

   "If I removed the sticky notes, can I still compare the number in each category?"
   "How could I show how many responses there are in each category?"

   Listen for suggestions that you could put numbers on a vertical line (axis). Ask:

   "What numbers do we need on the vertical line (axis)?"

   After discussion, place appropriate numbers on the vertical axis and highlight the fact that the choice of numbers (scale) depends on the data. Remove the sticky notes.



7. Ask:

   "What further information needs to be included on the graph so that a reader could understand the story it tells?"

   Listen for and encourage the response that the graph and each axis need a title.

   Discuss and insert appropriate titles.

8. Encourage interpretive discussion of the graph.

   For example, for the question "What do you do in your leisure time?" ask:

   "Which is the leisure activity that most of us do?"
   "What is the least common leisure activity?"
   "Do the responses agree with what we predicted?"
   "If there are differences from what we predicted, can you explain why?"

Follow-up activity

Ask the learners to write a short report for another group of learners on the original question, including some comments on how sure they are of what they are saying and why.

Sort, represent and interpret number data

Preparing Data for Analysis progression, 4th–5th steps; Analysing Data for Interpretation progression, 4th–5th steps; Interpreting Data to Predict and Conclude progression, 4th–5th steps

The purpose of the activity

In this activity, the learners sort and organise number data and represent it on graphs in order to develop the ability to read and derive meaning from graphs created by themselves or others.

The teaching points

- First drawing a bar graph to represent ungrouped data and then a graph of grouped data helps the learners understand graphs that have two sets of numbers.
- The grouped data graph gives information about the number of pieces of data, typical values and the spread of values.
- There are levels of deriving meaning from graphs. Ask the learners questions that will encourage them to think at each of these levels:
  - reading the data – lifting information from the graph
  - reading between the data – interpreting information in the graph
  - reading beyond the data – predicting or inferring from the graph
  - reading behind the data – connecting the information and its context.
- Discuss with the learners how the purpose of statistics is to predict and tell a 'story' and how when the learners are developing understanding of statistics it should always be in the context of a 'story' and that 'story' needs to be considered when preparing, analysing and interpreting data.

The guided teaching and learning sequence

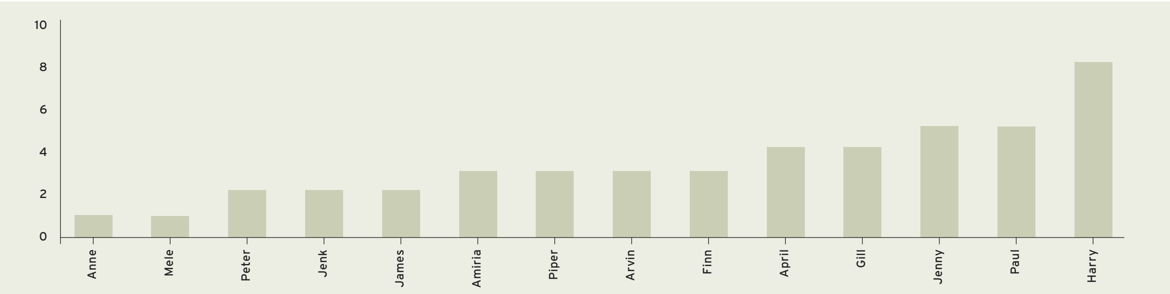
1. Choose a 'story to tell' that is of interest and relevant to the learners and that generates number data, for example, "How many children do we have in our families?".
2. Ask the learners:

   "How many children do we have in our families?" Issues will arise as to whether we are meaning the family in which we are the parents or in which we are the children.

   Ask:

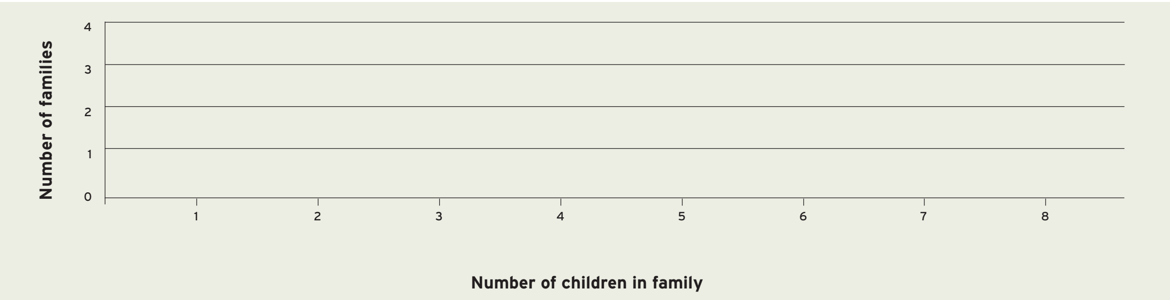
   "Is it important we all agree on what type of family we are talking about?" "Why?"
   "Who are we going to include for this exercise?"
3. Once you have determined the extent of 'family', ask the learners to identify how many children are in their family. Find out the largest number of children in the learners' families.
4. Draw a set of axes on the board with the vertical axis marked to the largest number. Include a title for the graph. Ask the learners to order their responses in terms of increasing family size and put their amount on the graph as a bar that is labelled along the horizontal axis with their name. (Note: this graph is representing the ungrouped data.)

Children in our families



5. Draw a second set of axes on the board.

Children in our families

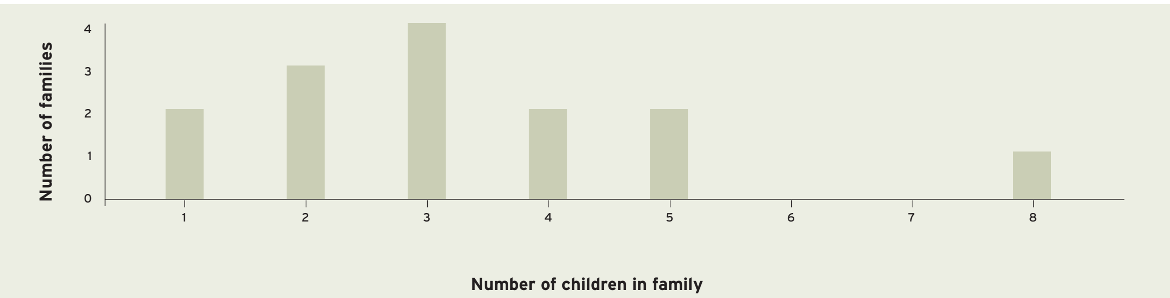


6. Referring to the graph that shows the ungrouped data, ask:

   "How many families have 1 child?" and draw a bar to show that amount on the second set of axes. "How many families have 2 children?" and draw a bar to show that amount on the second set of axes.

   Continue the questions until the graph showing grouped data is complete.

Children in our families



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7. Ask the learners to work in groups to discuss the questions you have prepared, using the four levels outlined in the teaching points above.

   For example:

   Reading the data

   "How many families have 3 children?"
   "What is the largest number of children in a family?"

   Reading between the data

   "How many families have 4 or more children?"
   "What percentage of families has 2 or less children?"

   (In this example, the answer is 36 percent (5 families out of 14) – encourage the learners to find this information from the grouped data graph, not just by counting the number in the class.)

   Reading beyond the data

   "If a new person joined our class, what do you think we could say about how many children would be in their family?"

   Listen for and encourage responses such as "almost certainly between 1 and 8 children" and "unlikely to be more than 8 children".

   The learners are also likely to comment on more families having 2 and 3 children. Discuss with them that this is the case for your class but that small samples vary – the neighbouring class might be quite different – and therefore your class's results are not a good basis on which to make judgements.

   Reading behind the data

   "If you went to another country or asked the question in a primary school class or elderly citizens' club, would you get the same result?"

Follow-up activity

Find interesting and relevant bar graphs from the newspaper or internet (for example, http://www.censusatschool.org.nz/2005/data-viewer/) and prepare questions that relate to the four levels of deriving meaning for the learners to assess in relation to the graphs.

Understanding the mean I

Analysing Data for Interpretation progression, 6th step

The purpose of the activity

In this activity, the learners develop an understanding of the concept of a mean as a number that represents what all the data items would be if they were levelled out to be the same. This activity is targeted to those learners who have no understanding of the mean or those who know the add-up-and-divide algorithm for the mean but do not understand why it works.

The teaching points

- The learners understand that measures that describe data with numbers are called statistics.
- The learners understand that both graphs and statistics can provide a sense of the shape of the data, including how spread out or how centred they are. Having a sense of the shape of the data is like having a big picture of the data rather than just having a collection of numbers.
- The learners understand that an average is a single number that is descriptive of a larger set of numbers. The mean, median and mode are specific types of average. Averages are measures of how centred the data is or the central tendency of the data.
  - The mean is computed by adding all of the values in the set and dividing the sum by the number of values added.
  - The median is the middle value of an ordered set of data. The median is relatively easy to compute and is not affected (like the mean is) by one or two very large or very small values that are outside the range of the rest of the data.
  - The mode is the number or value that occurs most frequently in the data. This statistic is least useful as often the mode does not give a very good description of the set. For example, 9 is the mode in the following set of values: 1, 1, 2, 2, 3, 4, 9, 9, 9.
- Discuss with the learners relevant or authentic situations where it is necessary to understand 'mean' and how to calculate it.

Resources

- Cubes (preferably ones that link together – these are available from educational resource centres).
- Paper strips (about 40 centimetres long).
- Sticky tape.

The guided teaching and learning sequence

1. Have each learner cut a strip of paper to the length of their foot. Ask them to record their names and the length of their foot in centimetres on the strip. Put the learners into groups of four or eight and give each group a roll of sticky tape. The task is for each group to come up with a method for finding the mean (the typical length) without referring to the amounts written on the strips. Pose the question:

   "How can you use the strips of paper to find out the mean length of feet in your group? (You must not look at the length written on the strips.)"

2. If the learners do not suggest a solution, prompt them by saying that the mean length is the length if all the lengths were the same. It may help to describe the mean as 'the typical length'. One option is to join the foot-strips end to end and then fold the combined strip into equal parts so that there are as many sections as learners in the group. Another option is to cut one piece from each of the larger foot strips and attach it to the end of each smaller piece until all the pieces are the same length.

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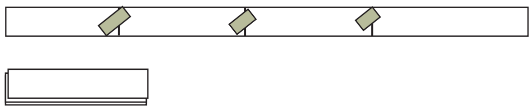
The learners can then measure the length of any one piece.



3. Ask the groups to share their approaches. Ensure that the learners understand that the length of the 'levelled' foot strip is the mean (typical length) of the foot lengths.

4. Next pose the question:

   "How can we find the mean for the whole class?"

   (Although it is possible to join the strips into a single length (say for 15 learners), it isn't practical to actually fold the strip into 15 equal parts.)

5. Ask the learners to think about how they could calculate the mean foot length without folding the long strip: "Without folding the strip, how can we find the mean (typical) foot length?" (The total length of the strip is the sum of the 15 individual foot strips. To find the length of one section, if the strip were actually folded in 15 parts, you divide by 15.)

6. Discuss with the learners how this process illustrates the usual add-up-and-divide algorithm for finding the mean.

7. Write the following meal prices for a group of 10 people on the board and ask the learners what they think everyone would have to pay if the group were to share the account.
   $12, $17, $17, $18, $19, $20, $22, $22, $23, $35

8. Ask:

   "What do you think the mean meal price per person would be?" "Why do you think that?"

   Listen for responses that confirm that the learners have understood the concept of 'levelling' to find the mean. In this case, the learners might see that $10 could be taken from $35 and put onto $12 to make $25 and $22 respectively. It then seems reasonable for the mean price to be around $20 or $21.

9. Next ask the learners to use the add-up-and-divide method to work out the mean price. Ask:

   "If the restaurant will only allow one account per table, exactly how much will each person pay if they all agree to pay the same amount?" (12 + 17 + 17 + 18 + 19 + 20 + 22 + 22 + 23 + 35 = 205; 205 ÷ 10 = $20.50)

   Note: Although the learners may use a calculator to sum the amounts, encourage them to mentally divide the total by 10.

10. Depending on your learners' level of understanding you may want to work through one or two more examples as a class before asking them to solve problems independently. Possible examples for further guided learning or independent work are:

    - mean meal cost: $16, $16, $19, $19, $21, $22, $25, $26, $28
    - mean daily temperature for the week: 16°, 17°, 23°, 18°, 18°, 19°, 20°.

    Before the learners calculate the mean in each instance, encourage them to apply the 'levelling' approach to obtain an estimate of the mean. Alternatively ask them to reflect on the calculated mean, asking if the answer they have obtained is 'reasonable'.

Follow-up activity

Pose problems that require the learners to find the mean of a set of values. Examples could include mean temperature, mean test score and mean height.

Understanding the mean II

Analysing Data for Interpretation progression, 6th step

The purpose of the activity

In this activity, the learners develop an understanding of the concept of the mean as the 'balance point' of a set of data. This concept describes the mean as the point on a number line where the data on either side of the point is evenly balanced. This activity builds on the "Understanding the mean I" activity and is targeted at those learners who understand the add-up-and-divide algorithm for finding the mean but do not understand the mean as a measure of the centre of a set of data.

The teaching points

- The learners understand that measures that describe data with numbers are called statistics.
- The learners understand that both graphs and statistics can provide a sense of the shape of the data, including how spread out or how centred they are. Having a sense of the shape of the data is like having a big picture of the data rather than just having a collection of numbers.
- The learners understand that an average is a single number that is descriptive of a larger set of numbers. The mean, median and mode are specific types of average. Averages are measures of how centred the data is or the central tendency of the data.
  - The mean is computed by adding all of the values in the set and dividing the sum by the number of values added. The mean is affected by very large or very small values (outliers) that are outside the range of the rest of the data.
  - The median is the middle value of an ordered set of data. The median is relatively easy to compute and is not affected (like the mean is) by one or two very large or very small values that are outside the range of the rest of the data.
  - The mode is the number or value that occurs most frequently in the data. This statistic is least useful as often the mode does not give a very good description of the set. For example, 9 is the mode in the following set of values: 1, 1, 2, 2, 3, 4, 9, 9, 9.
- Discuss with the learners relevant or authentic situations where it is necessary to have an understanding of the mean and how to calculate it.

Resources

- Sticky notes.

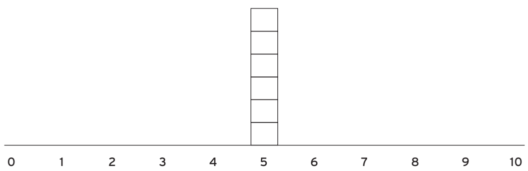
The guided teaching and learning sequence

1. Discuss with the learners the understanding that both graphs and statistics can provide a sense of the shape of the data, including how spread out or how centred they are. Tell the learners that some statistics measure the spread of data and others measure how centred the data is.
2. Ask the learners to explain what they think the mean of a set of numbers tells you. It is likely that the learners will have a variety of understandings of the concept of a mean including some or all of:
   - adding up the numbers and dividing by how many numbers there are (while this is an accurate description of an algorithm used to find the mean, it does not describe what the mean actually is)
   - the middle of the numbers (this is an ambiguous definition as it could describe the mean, the median or the mid-point between the highest and lowest values)

continued...

- the most common number (this is the mode, not the mean).

3. Tell the learners that this session develops an understanding of the mean as a balance point. Explain that the mean is one measure of the centre of a set of data.
4. Draw a number line on the board and label the points from 0 to 10. Place 6 sticky notes one above the other above the number 5 on the number line:

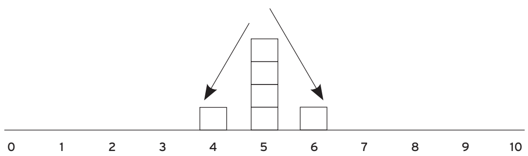


Ask:

"What is the balance point value on this number line?"
"How do you know?"

The learners may only see the mean as the result of adding up all the values and dividing by the number of values. This activity aims to encourage them to see the mean as the balance point. The mean of a set of values that are all the same must be that value.

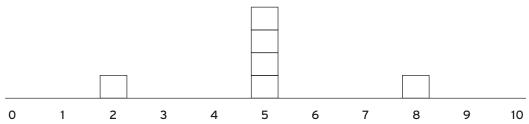
5. Ask the learners to suggest a way that the data could be changed so the balance point value remains as 5. They are likely to suggest moving two of the sticky notes in opposite directions, so that one is placed at 4 and the other at 6. This will result in a symmetrical arrangement.

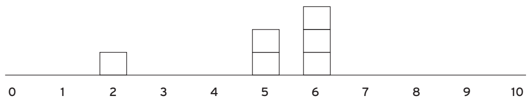


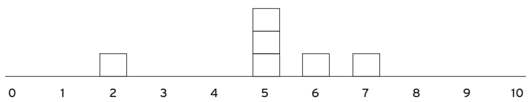
6. Put the sticky notes back on the 5 and ask:

"How many ways could you keep the balance point at 5 if we move one of the values to 2?"

Encourage the learners to see that a move of three spaces to the left could be balanced by moving one value three spaces to the right, or by moving three values one space to the right, or by moving one value one space to the right and one value two spaces to the right. Each time a balanced move is made you have made a new distribution with the same mean.







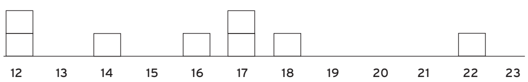
Help the learners notice that the mean only defines a 'centre' of a set of data and so by itself is not a very useful description of the shape of the data. The balance approach clearly illustrates that many different distributions can have the same mean.

7. As the balance concept does not lead directly to the add-up-and-divide algorithm for computing the mean, it is important to link the result of the add-up-and-divide algorithm to the 'balance' that was found by moving the sticky notes. Discuss how the balance point approach can be used to find the mean of a set of numbers. Provide pairs of learners with the following sample data set and ask them to model this data set on a number line with sticky notes. Costs of meals: $14, $22, $17, $18, $17, $16, $12, $12.

8. Ask the learners to predict where the mean might be. Encourage them to avoid using the add-up-and-divide algorithm, and instead have them think about the 'balance point' of the meal prices.

   "What do you think the mean cost for these meals is?" "Why?"

9. Ask the learners to determine the actual mean by moving the sticky notes in towards the centre. For each move of a sticky note one space to the left, there must be a corresponding move one space to the right. Encourage the learners to move sticky notes multiple spaces or move multiple sticky notes at one time.



   "What is showing when you have moved all the sticky notes to one point?" (16 is the balance point or the mean.)

   Ask for volunteers to show how they moved the sticky notes to find the balance point (mean) of 16.

10. Depending on your learners' level of understanding, you may want to pose another problem for them to solve by moving sticky notes to find the balance point (mean).
    Cost of meals: $21, $24, $27, $25, $25, $25, $29, $29, $30, $35 (mean = $27)

    Before they move the sticky notes, ask the learners to estimate where they think the mean of the data set will be. Notice if they are estimating where the balance point or 'centre' of the data could be.

11. Link the 'balance' that was found by moving the sticky notes to the result of the add-up-and-divide algorithm:
    (21 + 24 + 27 + 25 + 25 + 25 + 29 + 29 + 30 + 35 = 270; 270 ÷ 10 = 27)

    Although the learners may use calculators to add up the values, encourage them to do the division by 10 mentally.

12. Write the following prices on the board, explaining that they are the cost of items in a 'lucky dip': $8, $12, $3, $5, $7, $1. Ask:

    "What is the mean cost of the items?"
    (36 ÷ 6 = $6)
    "What would happen to the mean cost if the $1-item were removed?" (The mean cost would increase significantly.)
    "What would be the new mean cost?" ($7)
    "How did you calculate it?"

    Suppose that one new item were added to the original set of prices that increased the mean from $6 to $8. What would be the cost of the new item? Ask the learners to share their strategies for calculating the cost of the new item. (One solution is that you know that the total price for 7 items with a mean cost of $8 would be 7 x 8 = $56. The new item must therefore be 56 – 36 = $20.)

Follow-up activity

Ask the learners to create 'mean' problems for others to solve. Expect the learners to use the add-up-and-divide algorithm to compute the mean. Encourage the learners to reflect on the calculated mean, asking if the answer they have obtained is 'reasonable' in relation to where the 'balance point' of the data is.

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Sample sizes

Analysing Data for Interpretation progression, 4th–5th steps

The purpose of the activity

In this activity, the learners develop an understanding of how sample size influences the accuracy of a survey.

The teaching points

- The learners understand the concept of variation, including: each sample gives different results; smaller samples tend to be more variable than larger samples; larger samples tend to be more similar to each other and to the population data.
- The learners understand that a random sample is a sample of a population where each member of the population has an equal chance of being in the sample.
- Discuss with the learners relevant or authentic situations where the size of the sample influences the accuracy of the results (quality checks, election polls).

Resources

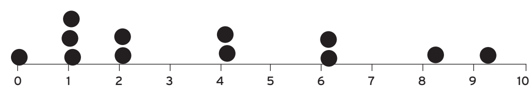
- Sets of 100 'glasses of water' cards (one set for each group of learners) (See Appendix B: Glasses of water cards).
- Dot plot template (See Appendix C: Dot plot template).
- Glasses of water graph and template (See Appendix A: Glasses of water graph).

The guided teaching and learning sequence

1. The context for this activity relates to the question "How many glasses of water do New Zealand tertiary learners drink in a day?" Discuss with the learners what they know about the benefits of drinking water and how many glasses of water they drink each day. Ask:

   "About how much water is it recommended we drink each day?"
   "What do we mean by a glass?"
   "Do other drinks count?"
   "Is it important we all agree on what to include as glasses of water?"

2. Record on the board the number of glasses of water drunk by people in the class the previous day. Here is an example set of responses from 12 people: 4, 4, 2, 6, 1, 0, 1, 6, 8, 2, 1, 9
3. Illustrate these results on a dot plot:



   "What can you say about this information?"
   (More than half drink 4 or less glasses of water a day; only 2 people (17%) drink 8 or more glasses a day)
   "What is the mean?"
   "What is the median?"
   "What is the range?"
   "Can we use the results of this survey to make comments about New Zealand tertiary learners more generally?"

4. Ask the learners how we could improve the survey to answer the question: "How many glasses of water do New Zealand tertiary students drink in a day?"
   The learners' responses may bring up a number of issues, including:
   - The class is not representative of all tertiary learners.
   - The sample was not random.
   - The sample size is only 12, which is not large enough.
   - The previous day is not random and may have been affected by weather or activity.
5. The size of the sample raises the question of how large a sample should be. To investigate this further, explain to the learners that they are going to compare samples taken from a 'population'. Have the learners work in groups or pairs and give each group or pair a bag of 100 glasses of water cards. Tell them that the bag contains 100 people's responses to the glass of water survey. Ask them to take 10 cards from the bag and record the results on a dot plot.
6. Tell them to repeat this four more times, returning the 10 cards to the bag between each sample. Give the learners the dot plot template (Appendix C) so that the dot plots are recorded under each other for comparison. Ask the groups to consider their five dot plots. Ask:

   "Do all your graphs (dot plots) have the same shape?"
   "What do you think the next sample of 10 will be?"
   "Can you confidently say how much water you think the population of 100 people drink?"
7. Encourage the learners to notice that each sample is different from all the others and that there is considerable variation in small-sized samples. Encourage the learners to use statistical language when they are describing the dot plots by writing key words ('sample', 'population', 'sampling distribution', 'outliers', 'spread', 'range', 'skew') on the board for the learners to refer to.
8. Ask the learners to take 30 cards from the bag and record the results on a dot plot. Display the dot plots from the various groups of learners on the board. Ask the learners to make statements about the graphs. To encourage more discussion about the shape of the graph, you could show the following sketches of graph shapes and ask the learners which drawing is the best shape for their data.

   "Which is the closest shape to your data?"
   "Which graph is least like your data?"
   Encourage the learners to explore their reasoning.
9. Ask the learners to display their data on a bar chart, using the model from Appendix A. Discuss the fact that the use of bar charts (with percentages) rather than dot plots (with frequencies) allows samples of different sizes to be compared more easily.
10. Display the graph of the population (Appendix A: Glasses of water graph) and ask the learners to reflect on how close their samples of 30 and 10 were to representing the population.

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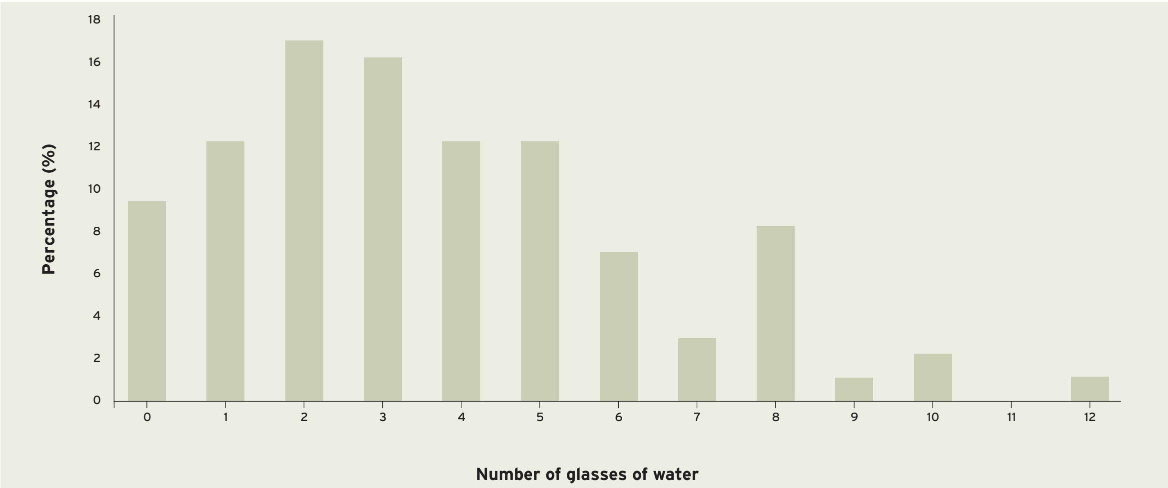
Number of glasses of water per day



11. Ask:

    "Which of the samples (10 or 30) were closest to representing the population?" (Generally smaller samples tend to be more variable than larger samples. This means that small samples are often not representative of the population.) "Were all the samples of 30 equally close at representing the population?" "Why or why not?" (Larger samples tend to be more similar to each other and to the population data; however, they will still be variable.)

Follow-up activities

Ask the learners to look for reports of surveys in local newspapers. Ask them to consider if they have enough information to be confident about whether the survey is representative of the population, considering the following questions:

"What is the size of the sample?"
"How was the sample selected?"

Understanding media reports that include statistical information

Interpreting Data to Predict and Conclude progression, 6th step

The purpose of the activity

In this activity, the learners develop the ability to interpret and critically analyse a media report that includes statistical information. This ability requires the learner to integrate their knowledge and understanding of general literacy, including numeracy, statistical ideas and critical skills.

The teaching points

- The learners need to be familiar with the context of a report in order to interpret and analyse it.
- The learners need to develop a critical approach to media reports that include statistical information. This includes:
  - being aware that credible evidence is a better basis for forming opinions than personal experience or anecdotal evidence
  - having some knowledge of methods used to collect information such as census, survey, experimental and control group information, and understanding that how the information, is collected impacts on its reliability
  - understanding the difference between a sample and the population and understanding that the size of the sample and the way it is chosen impacts on the reliability of information collected
  - understanding that information needs to be summarised to show key features but that this may hide variation in the information
  - being aware that ignoring simple conventions for displaying information (some examples are offered at http://lilt.ilstu.edu/gmklass/pos138/datadisplay/badchart.htm) can distort information, and this may be done deliberately.
- Guideline questions can help the learners develop a critical approach:
  - Is there information given about:
    - what type of study it is (based on samples from the census, experiments, etc)?
    - who or what was studied, how many and how they were chosen?
    - who collected the data and how?
    - how the information gathering came about and who funded it?
  - If this information is given, are the answers to the questions such that you can have confidence in the results?
  - Is the information presented clearly, using conventions for tables and graphs?
  - Is the information complete, does it make sense and does it agree with what you already know?

Resources

- One copy of a media report for each learner.

The guided teaching and learning sequence

1. Choose a media report that is of interest and relevant to the learners (and possibly controversial). Begin the session by asking the learners what they already know about the subject of the report. Listen for their opinions on the topic and differences in opinion, and ask the learners to consider how they formed their opinions. Challenge the learners to consider the fact that evidence is a better basis for forming decisions than their personal experience.
2. Divide the learners into groups and ask them to read and discuss the media report. Ask each group to prepare a brief summary of what the report is saying. Encourage the learners to identify in the summary the aspects of the report they used as the basis for their interpretations.

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3. Ask the learners to share their summaries with the whole class. Encourage the learners to look carefully at any graphs and tables to ensure they follow accepted conventions. Discuss any statistics reported and what they might or might not indicate about the underlying data.
4. Once broad agreement has been reached on what the report is saying, ask the learners to discuss how sure they are that the findings reported in the article are correct. Continually challenge the learners to give reasons for the opinions they express.

   Listen for discussion, and prompt if necessary, about whether information is given in the article about:

   - how the information was collected
   - who or what was studied, how many and how they were chosen
   - who collected the information and how it was funded.

   If such information is not included in the report, ask the learners to discuss whether they can have confidence in the findings.

   If the information is included in the report, ask the learners to discuss whether the manner in which the information was gathered enables them to have confidence in the findings.
5. Ask the learners to return to their groups and discuss if there are any changes they wish to make to their summaries of the article, following the discussion. Ask the groups to share any changes.
6. Conclude the session by asking the learners to recall the original discussion about the topic. Ask the original question again and ask the learners to share whether their views have been changed or strengthened by the media report. Encourage the learners to challenge each other to provide evidence for the views expressed.

Follow-up activity

Ask the learners to individually, and then as a group, prepare a list of questions that could be used to critically analyse media reports.

Describing likelihood

Probability progression, 1st–2nd steps

The purpose of the activity

In this activity, the learners develop an understanding of the words used to describe the probability or likelihood of events. The learners also learn to identify all the possible outcomes for a simple probability event.

The teaching points

- The learners understand that the probability that a future event will occur can be described along a continuum from impossible to certain.
- The terms that can be used for probability include "chance", "likelihood", "odds", "percentage" and "proportion".
- A sample space is the set of all possible outcomes for an event. For example, the sample space for drawing balls from a bag that only holds black and white balls is {black, white}.
- Discuss with the learners the fact that the outcomes of events are not usually equally likely. For example, the possible outcomes for a basketball free throw are either to make the goal or miss it, with the likelihood of making it dependent on the skill of the player. On the other hand, tossing a fair coin does have two equally likely outcomes.

Resources

- Two colours of counters, beads or paper (For the remainder of this sequence, we assume that black and white are the two chosen colours.)
- Non-transparent bags or containers.

The guided teaching and learning sequence

1. Begin the session by drawing a line on the board and writing the words "Impossible" and "Certain" at either end of the line.

2. Have the learners work in pairs or small groups. Give each group 10 black counters, 10 white counters and a container. Explain that you want them to put counters into a container so that the chance or probability of getting a black counter is certain.
3. Ask one of the groups to hold up their container for you to select a counter from. Before revealing the counter, ask the group to confirm that they are certain that you have a black counter in your hand.

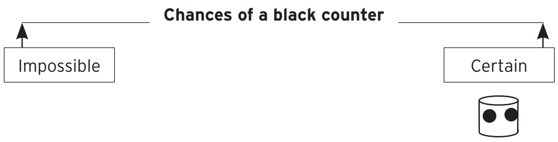
   "Are you certain that I have selected a black counter?" "Why?"
4. Hopefully you have a black counter in your hand and only black counters were in the container. Discuss with the learners whether they think that the number of black counters placed in the container makes a difference to the outcome.

   "Does it make a difference how many black counters you have in the container?"
   (No, because the only possible outcome is black and it doesn't matter whether there is 1 or 100 counters in the container.)
5. Ask the class to explain what it means to be certain about an outcome. Listen to ensure they understand that being certain means there is no chance of any other outcome.

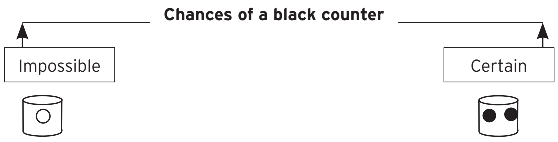
continued…

6. Under "Certain" on the probability line, draw a picture of a container with a number of black counters and record beside it the statement

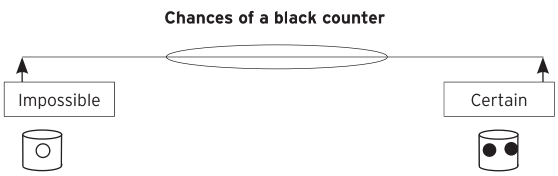
   "It is certain that you will draw a black counter from this container".



7. Ask the learners to put counters into the container so that the chance or probability of drawing a black counter is impossible. Record "It is impossible to draw a black counter from this container". Ask for volunteers to draw their container under "Impossible" on the probability line.



8. If necessary, explain that the container can only have white counters or no counters in it for this to happen and that the number of white counters does not influence the outcome.
9. Draw a circle around the centre of the line and ask the learners for words that describe that area on the line. Record suggested words on the board, looking for words such as: "likely", "equally likely", "even chance", "possible", "maybe", "probably".



10. Ask the learners to put counters in their container so that it is "possible" that you will get a black counter. Ask the learners to draw their container on the board.
11. Ask the learners to tell you what the possible outcomes are for each container. (There are two possible outcomes: black counter, white counter.)
12. Tell the learners that you have a container that holds 1 black and 100 white counters. Once more, ask the learners to describe the chance of taking out a black counter from this container and where that chance sits on the probability line. Possible descriptors include: "very unlikely", "highly unlikely", "not likely", "almost impossible", "poor chance", "not a very good chance", "a weak chance". Check that the learners understand that while it is highly unlikely it is not impossible, and therefore the container should be place closed to but to the right of "Impossible".

Follow-up activity

Ask the learners to work in pairs to draw a line and label each end "Impossible" and "Certain". Then have them record (in words or drawings) examples of events that are either impossible or certain.

Understanding chance

Probability progression, 3rd step

The purpose of the activity

In this activity, the learners further develop their concept of chance by discussing the likelihood of different events. They are introduced to ways of identifying all possible outcomes of a simple event (the event or sample space) and use fractions to assign likelihoods to outcomes.

The teaching points

- The probability that a future event will occur can be described along a continuum from impossible to certain.
- The probability of an event is a measure of the chance of an event occurring. The probability of an event is a number between 0 and 1 and can be expressed as a fraction, as a percentage (0–100 percent), or as odds.
- A sample space is the set of all possible outcomes for an event. For example, there are 6 possible outcomes for rolling a standard six-sided dice and 36 (6 x 6) possible outcomes for rolling two standard six-sided dice.
- The outcomes for events are not usually equally likely. For example, the possible outcomes for a basketball free throw are either to make the goal or to miss it, with the likelihood of making it dependent on the skill of the player. On the other hand, tossing a fair coin does have two equally likely outcomes.
- There are two ways to measure chance. One way is to analyse the situation logically (theoretical probability), and the other way is to generate data to analyse the situation (experimental probability). Examples of situations that can be analysed theoretically include rolling dice, throwing coins and Lotto. Examples of situations that need to be analysed experimentally include the likelihood of there being an earthquake or a car accident.

Resources

- A die for each learner or pair of learners.
- A large die for demonstration (if available).

The guided teaching and learning sequence

1. Begin the session by showing the learners a die and asking them which number they think will come up if you roll it.

   "What number do you think I will roll?"
   "Why do you think that?"

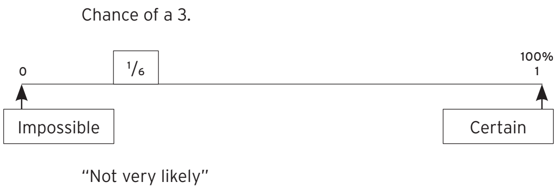
2. Check that the learners' responses show that they understand that it isn't possible to predict what will happen when you roll a standard six-sided die.
3. Roll the dice and see if the learners guessed correctly.
4. Ask the learners to state the possible outcomes for rolling the dice. List these on the board.
   What are the possible numbers that I can roll?
5. Tell the learners that this list of all the possible outcomes is called the sample or the event space: {1, 2, 3, 4, 5, 6}
6. Draw a line on the board, labelling one end "Impossible" and other "Certain". Ask the learners if they know which numbers should go with the words. (0 and 1 or 0 and 100%.)
7. Record these on the line.

8. Ask:

   "What is the chance of rolling a 3?" ($^1/_6$)
   Ensure the learners understand that there is 1 chance in 6 of rolling a 3 and that this can be expressed as one-sixth or $^1/_6$.

continued...

9. Ask a learner to put a mark on the line at $1/6$. Check that the learners understand that this mark is $1/6$ of the way along the line. Also ask the learners to suggest words that could be used to describe the chance of rolling a 3.



10. Ask:

    "What is the chance of getting an even number when you roll the dice?" ($1/2$ or $3/6$ or 50%).

11. Ask the learners to explain their reasoning. Check that they understand that there are two possible outcomes in the sample space {even numbers, odd numbers} and that the two outcomes are equally likely because three of the numbers are odd (1, 3, 5) and three are even (2, 4, 6). Check also that the learners understand that $3/6$ is equivalent to $1/2$ and 50%.

12. Record $1/2$ on the line and ask the learners to suggest words to describe this probability, for example, "equally likely" or "even chance".

13. Ask:

    "What is the chance of getting a number greater than 2?" ($4/6$ or $2/3$)

14. Ask the learners to explain their reasoning. Check the learners understand that there are two outcomes in the sample space. One is numbers greater than 2 (3, 4, 5, 6) and the other is numbers 2 or less (1, 2).

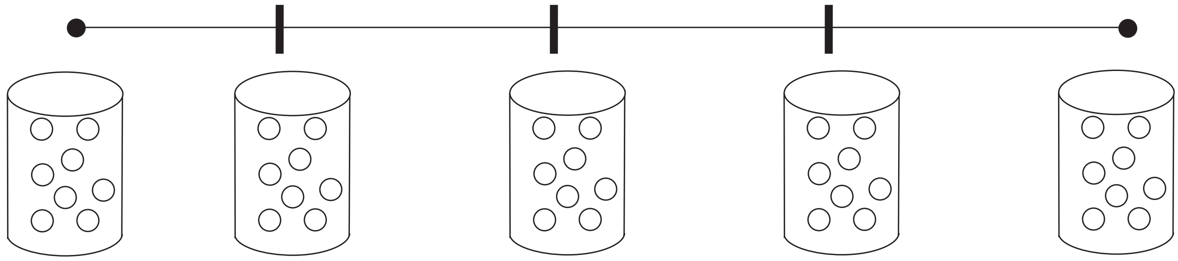
15. Ask the learners to suggest events for rolling the dice that are:

    - certain (for example, rolling a number less than 7)
    - impossible (for example, rolling a number greater than 6).

Follow-up activity

Give the learners a copy of the following diagram. Explain that they are to:

1. Shade the counters shown in the containers so the chance of drawing a shaded counter matches the probability marker on the line.
2. Label the markers with the fraction that represents the probability of drawing a shaded counter.
3. Write words with each marker to describe the probability.



Using frequencies to predict

Probability progression, 4th-5th steps

The purpose of the activity

In this activity, the learners use the frequencies of outcomes to predict the likelihood that an event will occur. The activity reinforces the idea that probabilities are not absolute predictors in the short run.

The teaching points

- The probability that a future event will occur can be described along a continuum from impossible to certain.
- The probability of an event is a measure of the chance of an event occurring. The probability of an event is a number between 0 and 1 and can be expressed as a fraction or a percentage (0–100 percent).
- A sample space is the set of all possible outcomes for an event. For example, there are six possible outcomes for rolling a single standard six-sided die.
- The outcomes for events are not usually equally likely. For example, the possible outcomes for a basketball free throw are either to make the goal or to miss it, with the likelihood of making it dependent on the skill of the player. On the other hand, tossing a fair coin does have two equally likely outcomes.
- There are two ways to measure chance. One way is to analyse the situation logically (theoretical probability), and the other way is to generate data to analyse the situation (experimental probability).
- Discuss with the learners examples of situations that can be analysed theoretically, including rolling dice, throwing coins and Lotto, and examples of situations that need to be analysed experimentally, including the likelihood of there being an earthquake or a car accident.

Resources

- Small plastic cups.
- A saucer.
- A coin.

The guided teaching and learning sequence

1. Tell the learners that you are going to toss a fair coin. Ask those who think it is going to land as heads to stand up. Assuming that there are roughly equal numbers standing and sitting, ask: "Why do you think about half of you are standing and the others sitting?" (Check that the learners know there are two outcomes and that they are equally likely.)
2. Toss the coin, noting the outcome.
3. Ask:

   "If I tossed the coin one hundred times, how many heads do you think I would get?"
   "Why do you think that?"
   "Do you think that I will get exactly 50?"
   "Why or why not?"

   (You can expect about 50 heads and 50 tails, although anything in the range of 40–60 is reasonable.)
4. Next show the learners a small plastic cup and tell them that you are going to toss it in the air and let it land on the ground. Ask the learners to tell you the possible outcomes for the cup landing. Record these on the board (upside down, right way up, on side).

continued...

5. Remind the learners that this list of outcomes is called the sample space.

   "Do you think that the 3 outcomes are equally likely?" "Why or why not?"
   "Which of these outcomes do you think is most likely?"
   "Can you estimate the probability of the cup landing the right way up?"

6. Ensure that the learners understand it isn't possible to analyse the cup-tossing event mathematically and that you need to conduct an experiment to estimate the probabilities of the three possible outcomes.

7. Have the learners work in pairs and give each pair a small plastic cup and tell them that they are going to toss it 20 times and record how it lands each time. Before they begin tossing the cup, encourage the learners to agree on a uniform method for tossing the cup (for example, standing up, flipping the cups to the same height, letting the cups land on the floor or on a table).

   Upside down: 2
   Right way up: 1
   On side: 17

8. Ask the learners to share the results of their 20 trials. Discuss the differences and generate reasons for them. Possible reasons include: samples (cups' size and shape) may vary, cup-tossing techniques may vary.

9. Ask the learners how they could use the data to estimate the probability for the three cup-tossing outcomes. In the discussion, work towards ensuring the learners understand that a more accurate prediction can be made if the results of the class's trials are combined.

10. Combine the class's trials and use this data

    Upside down: 12
    Right way up: 7
    On side: 150

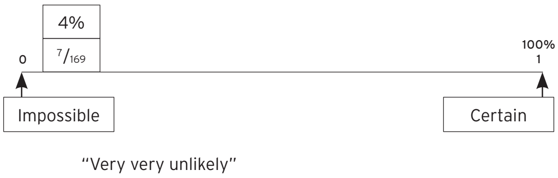
    to give a better estimate of the probability of the three outcomes, using fractions and percentages.

    Check the learners understand why the fractions (and percentages) when added together equal 1 (or 100%).

	CLASS TRIALS	PROBABILITY (FRACTION)	PROBABILITY (%)
Upside down	12	$^{12}/_{169}$	7%
Right way up	7	$^{7}/_{169}$	4%
On side	150	$^{150}/_{169}$	89%

Note: This activity also gives you the opportunity to ensure the learners understand how to calculate percentages from fractions.

11. In summary, draw a line labelled at either end 0 and 1 respectively (100%). Ask for volunteers to locate the three outcomes on the line and to give a language-based description of each outcome.



This activity also provides an opportunity to check the learners understand the relative size of various fractions and percentages and whether they understand which events are very unlikely or moderately or highly likely.

12. Discuss with the learners real-world situations where the probabilities need to be calculated from frequencies (insurance company premiums, weather forecasting, the chance of getting breast cancer, etc). Ensure the learners understand that one limitation of the relative frequency approach is that the probabilities are only estimates because they are based on finite samples of collected data.

Follow-up activity

Give each learner (or pair of learners) two counters and a saucer. Ask the learners to calculate the probability of flicking a counter into the saucer from a distance of 10 centimetres. As this is a skills-related activity, each learner's probability of success will be different.

Theoretical probability

Probability, 4th-5th steps

The purpose of the activity

In this activity, the learners develop their understanding of the difference between theoretical and experimental probabilities.

The teaching points

- The probability of an event is a measure of the chance of an event occurring. The probability of an event is a number between 0 and 1 and can be expressed as a fraction, a percentage (0–100%), or as odds.
- A sample space is the set of all possible outcomes for an event. For example, there are 6 possible outcomes for rolling a single standard six-sided die and 36 (6 x 6) possible outcomes for rolling two standard six-sided dice.
- There are two ways to measure chance. One way is to analyse the situation logically (theoretical probability), and the other way is to generate data to analyse the situation (experimental probability). Examples of situations that can be analysed theoretically include rolling dice, throwing coins and Lotto. Examples of situations that need to be analysed experimentally include the likelihood of there being an earthquake or a car accident.

Resources

- A series of three different-coloured counters.
- A non-transparent bag.
- Coins.

The guided teaching and learning sequence

1. With the learners watching, put 10 coloured counters in a bag: 5 red, 3 blue and 2 yellow. Record the colours and numbers of counters placed in the bag on the board for the learners to refer to.
2. Ask the learners to state what will happen if a counter is selected at random from the bag:

   "What colour counter will be selected?"
   "What colours of counter could be selected?"
   "What colour is most likely?" "Why?"

   Some learners are likely to try to tell you that a red counter will be chosen. It is important for them to realise that they cannot be certain what colour will be selected. If one is chosen at random, it could be any of the three colours. Red is more likely than blue or yellow because there are more red counters than blue or yellow counters in the bag.
3. Now ask the learners to describe the chance of selecting a counter of each colour.

   "What is the chance of selecting a red, blue or yellow counter?" "How did you work that out?"

   The learners should be able to state:

   - There is a 50% or $^1/_2$ chance of selecting a red counter because 5 of the 10 counters are red.
   - There is a 30% or $^3/_{10}$ chance of selecting a blue counter because 3 of the 10 counters are blue.
   - There is a 20% or $^1/_5$ chance of selecting a yellow counter because 2 of the 10 counters are yellow.
4. Explain to the learners that these are the theoretical probabilities of the event.

5. Discuss what these theoretical probabilities mean in terms of selecting counters. For example, you are twice as likely to select a red counter as any other colour.
6. Ask the learners what they think will happen if you select 10 counters, one at a time from the bag, returning the counter to the bag between selections. The learners are likely to suggest that there will be 5 red counters, 3 blue counters and 2 yellow counters selected. While this is possible, it is not the only possible outcome, in fact the probability of selecting exactly 5 red, 3 blue and 2 yellow counters is quite low.
7. Ask 10 learners to each come up, select a counter, record its colour and replace it in the bag. Graph the results on the board and discuss the results.
8. Repeat several times.
9. Ask the learners to discuss in small groups why the results do not match the theoretical probabilities.
10. Share the ideas as a class. Hopefully some learners will recognise that theoretical probability is only a way of predicting what *may* occur, it does not tell you what *will* occur. Ensure that all the learners understand this.

Follow-up activity

Ask the learners to work in pairs to calculate the theoretical probabilities for tossing two coins. Then ask them to toss two coins 100 times and compare the results of the experiment with their theoretical probabilities.

Theoretical probabilities:

- 2 Heads: $^1/_4$ or 25%.
- 2 Tails: $^1/_4$ or 25%.
- 1 Head, 1 Tail: $^1/_2$, 50%.

Lotto winners

Probability progression, 6th step

The purpose of the activity

In this activity, the learners explore issues related to gambling to further develop their concept of chance. They learn to use tree diagrams, organised lists and to multiply probabilities to calculate the theoretical probability of winning Lotto.

The teaching points

- The probability of an event is a measure of the chance of an event occurring. The probability of an event is a number between 0 and 1 and can be expressed as a fraction, a percentage (0–100%), or as odds.
- There are two ways to measure chance. One way is to analyse the situation logically (theoretical probability), and the other way is to generate data to analyse the situation (experimental probability). Examples of situations that can be analysed theoretically include rolling dice, throwing coins and Lotto. Examples of situations that need to be analysed experimentally include the likelihood of there being an earthquake or a car accident.
- Discuss with the learners the fact that theoretical probability is only a way of predicting what *may* occur, and it does not tell you what *will* occur.

Resources

- Playing cards or counters with numbers 1–10 written on them.
- A paper bag or non-transparent container.
- A tree diagram (see glossary, *Learning Progressions for Adult Numeracy*).

The guided teaching and learning sequence

1. Tell the learners that you are going to play a simple Lotto game that involves just three numbers (1, 2, 3) and that the game requires a volunteer to pull out two numbers from a bag.
2. Ask the learners to write down the two numbers they think will be drawn.
3. The Lotto draw can be simulated by putting playing cards (in this case 1, 2, 3) in a bag and asking a volunteer to select two cards at random.
4. Draw the table below on the board and record the number of 'winners' and 'losers' in the first row.

	NUMBER OF WINNERS (%)	NUMBER OF LOSERS (%)
3-number Lotto		

   "What percentage of the class won?"
   "Is this what you expected? Why or why not?"

5. Encourage the learners to analyse the theoretical probability of winning. In 3-number Lotto, there are three combinations (1 and 2, 1 and 3, 2 and 3) since the order of the draw does not matter. Therefore the probability of winning is 1 out of 3 or $^1/_3$ or 33.33%.
6. Discuss with the learners the methods that can be used to work out the number of possible outcomes. These include:
   - using a tree diagram
   - making an organised list:
     1, 2
     1, 3
     2, 3
   - $^2/_3 \times ^1/_2 = ^2/_6$ or $^1/_3$ (Note: There is a $^2/_3$ chance in relation to the draw for the first number and a $^1/_2$ for the second.)

7. Reflect back on the percentage of winners in the class simulation, ensuring the learners understand the theoretical probability is only a way of predicting what *may* occur, and it does not tell you what *will* occur.

8. Repeat steps 1–7 above with other versions of Lotto:

4-number Lotto	Select 2 numbers	$^1/_6$ chance of winning
5-number Lotto	Select 2 numbers	$^1/_{10}$ chance of winning
5-number Lotto	Select 3 numbers	$^1/_{10}$ chance of winning
6-number Lotto	Select 3 numbers	$^1/_{20}$ chance of winning

The key points to observe with the learners are that:

- the more numbers there are to choose from, the less the probability of a win
- the number of winners in the class (the experimental probability) is not usually the same as the theoretical probability.

9. When the learners understand how to calculate the probability of winning with the simple Lotto games, investigate the chance of winning first-division in Lotto (40 numbers, select 6).

"Which method can we use to find the number of possible outcomes?" (Hopefully the learners will realise that creating an organised list or using a tree diagram is not practical in this instance.)

Work together as a class to calculate the probability of winning first division:

$^6/_{40}$ chance of having the first number drawn
$^5/_{39}$ chance of having the second number drawn
$^4/_{38}$ chance of having the third number drawn
$^3/_{37}$ chance of having the fourth number drawn
$^2/_{36}$ chance of having the fifth number drawn
$^1/_{35}$ chance of having the sixth number drawn.

Thus there is a $^6/_{40} \times {}^5/_{39} \times {}^4/_{38} \times {}^3/_{37} \times {}^2/_{36} \times {}^1/_{35} = {}^{720}/_{2,763,633,600} = {}^1/_{3,838,380}$. (Note: this needs to be calculated on a computer as the number is larger than that permitted by most calculator displays.)

10. Discuss with the learners what a chance of 1 in 3.8 million means.

"What does a 1 in 3.8 million chance mean?"
"If you bought 10 lines a week would you be certain of a win in 380,000 weeks (and how many years is that)?"
"Would you make money if you bought enough lines to cover all the numbers?"
(no, it would cost 3,838,380 x 60 cents = $2,303,208)

Follow-up activity

Investigate the chance of winning a first-division Lotto strike, a first division Lotto Powerball, or Big Wednesday.

Note: The New Zealand Lotteries Commission website www.nzlotteries.co.nz has information on the different Lotto games, including printable documents.

Fair games?

Probability progression, 6th step

The purpose of the activity

In this activity, the learners explore issues related to gambling to further develop their concept of chance. They calculate theoretical probabilities of game outcomes and develop an understanding of expected value.

The teaching points

- The probability of an event is a measure of the chance of an event occurring. The probability of an event is a number between 0 and 1 and can be expressed as a fraction, a percentage (0–100%), or as odds.
- There are two ways to measure chance. One way is to analyse the situation logically (theoretical probability), and the other way is to generate data to analyse the situation (experimental probability). Examples of situations that can be analysed theoretically include rolling dice, throwing coins and Lotto. Examples of situations that need to be analysed experimentally include the likelihood of there being an earthquake or a car accident.
- The expected value of an event is the sum of the probability of each possible outcome of the event multiplied by the value (or pay-off) of the outcome. The expected value represents the average amount one 'expects' as the outcome of the chance event. A common application of expected value is in gambling. For example, a roulette wheel has 38 equally likely outcomes. A winning bet placed on a single number pays 35 to 1 (this means that you are paid 35 times your bet, and your bet is returned, so you get 36 times your bet).

  So, considering all 38 possible outcomes, the expected value of the profit resulting from a \$1 bet on a single number is: (\$35 x $^{1}/_{38}$) + (–\$1 x $^{37}/_{38}$), which is about –\$0.05. This means that the expected value of a \$1 bet is \$0.95 or that you lose, on average, 5 cents for every dollar bet.
- Discuss with the learners where expected value is used to determine values: insurance premiums, gambling pay-offs, sports betting.

Resources

- Cubes with 3 red faces, 2 green faces, and a blue face.

The guided teaching and learning sequence

1. Show the learners the cube with the coloured faces and write the following game rules on the board.
   - The dice is rolled.
   - You pay \$2 to play.
   - If the face is red, you lose the \$2 it cost you to play.
   - If the face is blue, you are paid \$5 (you win \$3).
   - If the face is green, you are paid \$3 (you win \$1).

Tell the learners that they will start with \$10 and have to stop playing if they run out of money. Play 10 rounds of the game with the class. Ask a volunteer to keep a running tally of the money won or lost.

GAME	RESULT	TOTAL
1	blue	13
2	red	11
3	red	9
4	green	10
5	etc	

2. Have the learners work in pairs and give each pair a cube with the same coloured faces as the previous cube and ask them to play a further 10 rounds of the game, keeping a tally of the money won and lost.
3. As a class, share the final outcomes of the games played.

   "How many were winners? How many losers?"
   "What was the largest winning total?"
   "What was the largest loss?"
   "Do you think this is a fair game to play?"
   "Why or why not?"
   "What is a fair game?" "How can you work out whether the game is fair or not?"

   (A fair game is one where there are equal chances of wining and losing. In this case, it is a fair game if the cost of playing is equal to the winnings.)
4. Discuss with the learners the concept of 'expected value'. Explain that the expected value of a probability situation is the sum of the probability of each possible outcome of the event multiplied by the value (or pay-off) of the outcome.
5. Ask:

   "What do you think the expected value of the cube game will be?" (Encourage the learners to link their answers to the class outcomes of playing the game.)
6. As a class, work out the expected value of the cube game:

   "What are the possible outcomes in the coloured-cube game?" (blue, green, red)
   "What are the chances of each of these happening?" (blue 1/6, green 1/3, red 1/2)
   "What is the chance of winning \$5?" (1/6)
   "What is the chance of winning \$3?" (1/3)
   "What is the chance of losing \$2?" (1/2)
   "What is the expected value of playing the game?" ($^1/_2$ (–2) + $^1/_3$ (1) + $^1/_6$ (3) = –1 + $^1/_3$ + $^1/_2$ = $^{-1}/_6$)
   "Is the game fair?" (no, in the end you lose 16.6 cents each turn you play.)
7. Ask the learners to work in pairs to invent similar dice games. Circulate as the learners are creating their games, asking them to explain if their games are fair or unfair.
8. Ask the learners to look at the games developed by other pairs. The challenge is to work out which of the games has the greatest expected value and which has the least.

Follow-up activity

Ask the learners to investigate whether the following game is fair. Two friends play a game where a single dice is rolled. Here is what happens for each result:

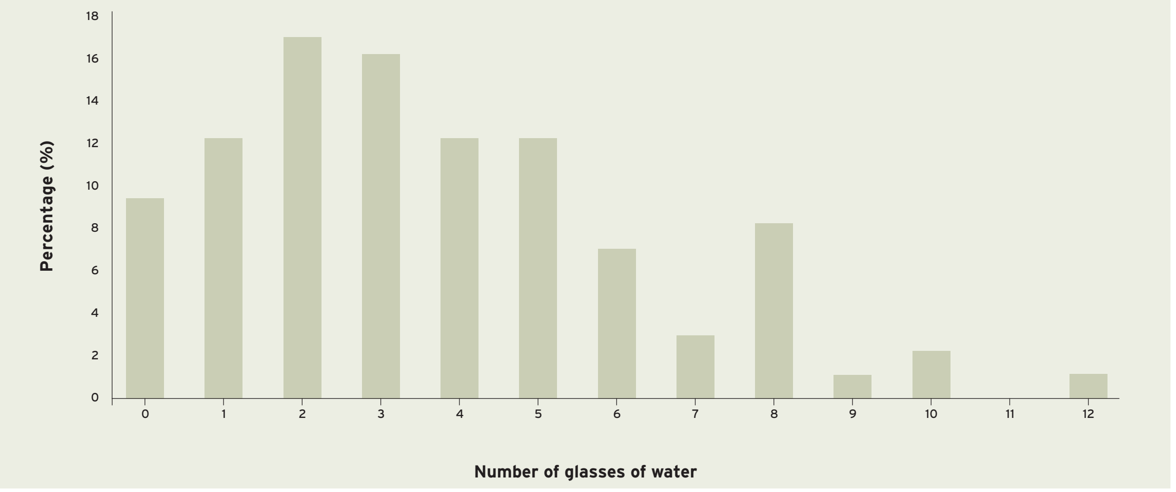
1: Player 1 wins \$3.
2: Nobody wins or loses.
3: Player 2 wins \$5.
4: Player 1 wins \$3.
5: Player 2 wins \$4.
6: Player 1 wins \$2.

"Is it a good idea to play the game?"
"If this game isn't fair, how could you change it to make it fair?

Appendices

Appendix A: Glasses of water graph

Number of Glasses of water per day



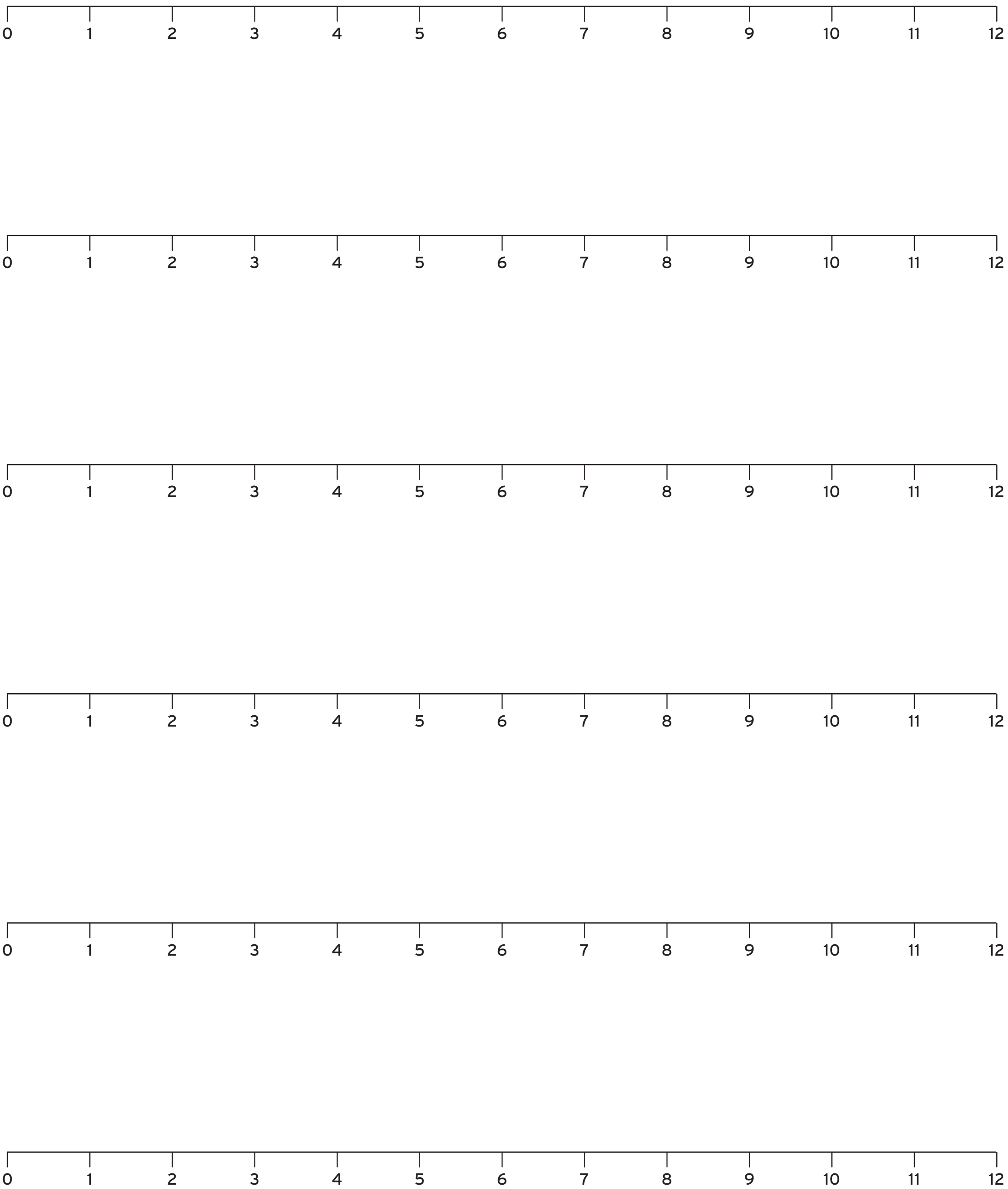
Appendix B: Glasses of water cards

Glasses of water cards (cut along lines)

0	0	0	0	0	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	3	3
3	3	3	3	3	3	3	3	3	3
3	3	3	3	4	4	4	4	4	4
4	4	4	4	4	4	5	5	5	5
5	5	5	5	5	5	5	5	6	6
6	6	6	6	6	7	7	7	8	8
8	8	8	8	8	8	9	10	10	12

Appendix C: Dot plot template

Dot plot template



Notes

Notes

Published 2008 by the Tertiary Education Commission.

Labour Force Status graph from: www.educationcounts.govt.nz/__data/assets/pdf_file/0019/16750/Factsheet_2-final.pdf

Data on page 7 sourced from: www.educationcounts.govt.nz/__data/assets/pdf_file/0019/16750/Factsheet_2-final.pdf

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Catalogue number TE185
ISBN 978-0-478-32000-8

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